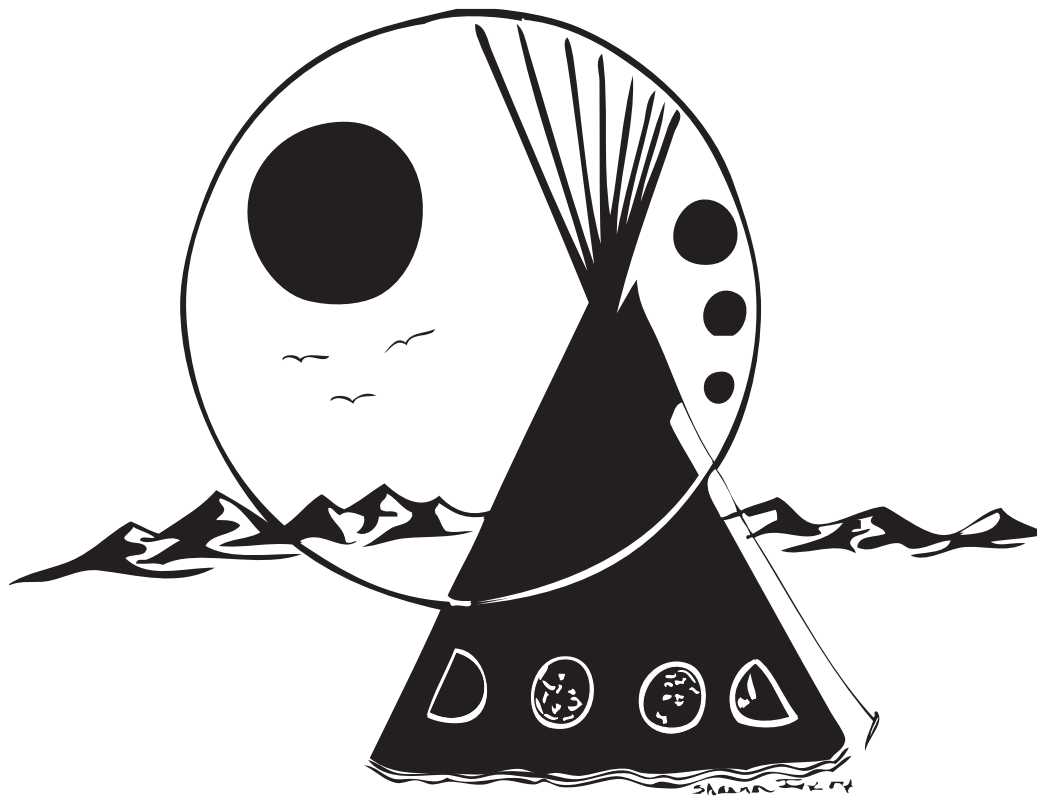


# Indian Education for All



## Model Teaching Units Mathematics - Grades 7-12

Developed by Montana Educators and  
the Office of Public Instruction



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Montana  
**Office of Public Instruction**  
Denise Juneau, State Superintendent

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Published 2010

Denise Juneau, Superintendent • Montana Office of Public Instruction • [www.opi.mt.gov](http://www.opi.mt.gov)



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# Mathematics Model Teaching Units Grades 7-12

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# Mathematic

## Model Teaching Unit

### Native American Designs

Created by: Lisa Scott

**Grade 7 – Approximate Duration: 135 minutes**

#### Stage 1 Desired Results

#### Established Goals:

**Geometric Reasoning Mathematics Content Standard 3:** A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant cultural contexts, including those of Montana American Indians.

- **3.3 Transformations including Dilations:** Define, identify, and execute transformations including translations, rotations, reflections, and dilations with appropriate technology.

**IEFA: Essential Understanding 1:** There is great diversity among the 12 tribal Nations of Montana in their languages, cultures, histories and governments. Each Nation has a distinct and unique cultural heritage that contributes to modern Montana.

**IEFA: Essential Understanding 2:** There is great diversity among individual American Indians as identity is developed, defined and redefined by entities, organizations and people. A continuum of Indian identity, unique to each individual, ranges from assimilated to traditional. There is no generic American Indian.

#### Understandings:

*Students will understand...*

- how to identify the different planar transformations and symmetries in Native American designs.
- how to use a geometry drawing utility to create their own design.

#### Essential Questions:

- How are the planar transformations of reflection, rotation and translation defined?
- How are the planar transformations constructed using a geometry drawing utility?
- How are line symmetry and rotational symmetry identified and defined?

*Students will be able to...*

- mathematically define reflection, rotation, translation, line of symmetry and rotational symmetry.
- describe and demonstrate each transformation and symmetry on a Native American design.
- construct a reflection, rotation and translation using a geometry drawing utility.
- identify and create a line of symmetry in a Native American design.
- identify and create rotational symmetry in a Native American design.
- create a design using a geometry drawing utility.

*Students will know...*

- the definitions of transformation, reflection, rotation, translation, line of symmetry and rotational symmetry.



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### Stage 2 Assessment Evidence

**Performance Tasks:** Students create their own designs. The design includes a reflection, rotation, translation, line of symmetry, and rotational symmetry. Designs are turned in and graded.

**Other Evidence:** Observation of identifying the transformations and symmetries. Participation in class demonstrations and discussions. Individual questioning of students.

### Stage 3 Learning Plan

#### I. Teacher Preparation: Create a PowerPoint

- a. The PowerPoint contains information which will be presented to students. Go to the Morning Star Gallery homepage, <http://www.morningstargallery.com/index.html>, to complete the preparation needed. Review images to use from the Morning Star Gallery. Select seven various images. Follow the format found in the example slide as a guide, duplicate as needed.

#### II. Learning Activities: Native American Designs

- a. Handout a copy of the PowerPoint presentation “Native American Designs” to each student. Discuss the definitions for a pre-image, image, reflection, rotation, translation, line of symmetry and rotational symmetry.
  - i. Have students work in pairs to identify any translations, reflections and rotations in each picture. Students should explain their reasoning for each transformation identified to each other and prepare to share the reasons during the class discussion.
  - ii. Have students find and draw line and rotation symmetries on the Native American designs. Students should explain their reasoning for each type of symmetry identified to each other and prepare to share the reasons during the class discussion.
  - iii. Project the PowerPoint onto the SMART Board (if available) or a whiteboard and have pairs take turns drawing in their transformations and symmetries. The pairs of students should explain their reasoning of each transformation and symmetry.
- b. Introduce a Geometry drawing utility such as Geometer’s Sketchpad, Cabri or GeoGebra. You can download GeoGebra for free from <http://www.geogebra.org>.
  - i. Demonstrate the drawing utility on the teacher computer while projecting with a LCD.
  - ii. Have students working individually or in pairs on a drawing utility during the demonstration.
  - iii. Encourage Free Play. Have students play with the free hand tools, point, compass, and segment. Use the selection arrow and text tool. Demonstrate how to undo objects.
  - iv. Demonstrate how to construct a polygon. Perform a reflection of the polygon over a line. Demonstrate the “click and drag” and how to change the color of the image. Show a reflection over a line that intersects the figure and a reflection over a line that does not intersect the figure by dragging the pre-image or image.
    1. Ask students to describe what they notice about the pre-image and the image as they click and drag points and sides of the pre-image or the image. (Students should notice that the pre-image and the image are always mirror images of each other over the line of reflection and the corresponding points of the pre-image and the image is always equidistant to the line of reflection.)
    2. If students have a difficult time seeing the pre-image and the image are always equidistant to the line of reflection, have them construct a segment from a point

on the pre-image and the corresponding image point. Construct the point of intersection between this segment and the line of reflection. Measure the distance from the pre-image point to the point of intersection between the segment and the line of reflection and measure the distance from the image point and the point of intersection between the segment and the line of reflection. From this construction, the students should see that the segment and the line of reflection are perpendicular to each other. This should lead students to writing a more formal definition for reflection. A **reflection** over a line is a transformation in a plane where each point of the original figure (pre-image) has an image that is the same distance from the line of reflection as the pre-image point. The line of reflection is the perpendicular bisector of the segment joining every point and its image.

- v. Perform a rotation of the constructed polygon around a marked point (center of rotation) by a fixed angle. Demonstrate the “click and drag” and change the color of the image.
  1. Ask students to describe what they notice about the pre-image and the image as they click and drag points and sides of the pre-image or the image. (Students should notice the corresponding points of the pre-image and the image is always equidistant to the center of rotation.)
  2. If students have a difficult time seeing the pre-image and the image are always equidistant to the center of rotation, have them construct a segment from a point on the pre-image to the center of rotation and the corresponding image point to the center of rotation. Measure the distance from the pre-image point to the center of rotation and measure the distance from the corresponding image point to the center of rotation. From this construction, the students should see that these segments are equal in length. Have students measure the angle formed by the segments drawn from the center of rotation to the pre-image and image points. Students should notice that this angle is equal to the angle of rotation they determined during the construction of the rotation. This should lead students to writing a more formal definition for a rotation. A **rotation** is a transformation in a plane that moves every point around a fixed point (usually the origin) in a given direction by a given angle measure. Rotations  $> 0^\circ$  are counterclockwise. Rotations  $< 0^\circ$  are clockwise.
- vi. Demonstrate a translation by a marked vector. Demonstrate the “click and drag” and change the color of the image.
  1. Ask students to describe what they notice about the pre-image and the image as they click and drag points and sides of the pre-image or the image. (Students should notice the distance between the corresponding points of the pre-image and the image is the same as the length of the vector and the image moves in the same direction the vector points.)
  2. If students have a difficult time seeing the distance between the corresponding points of the pre-image and the image is the same as the length of the vector, have them construct a segment from a point on the pre-image to the corresponding image point and measure the length of the segment. Measure the

*Mathematics Grade 7 – Native American Designs (continued)*

length of the vector. This should lead students to writing a more formal definition for a translation. A **translation** is a transformation in a plane that moves every point in the pre-image the same distance in the same direction.

- vii. Show how to hide objects that the students don't want seen but are necessary for the construction of the transformation.
- viii. Ask students to work on their designs. Circulate and help as needed.

**III. Summary:** Native American Designs are rich with transformational geometry. This lesson introduced three transformations. These included a reflection, a rotation, and a translation. This lesson also addressed the definition for a line of symmetry and rotational symmetry using Native American Designs.

**IV. Materials:**

- Teacher computer
- LCD projector and SMART Board (if available)
- PowerPoint of Native American Designs
- Computers or calculators with Geometer's Sketchpad, Cabri or GeoGebra for each student or pairs of students
- Copies of Native American Designs PowerPoint pgs. 1-10 for each student
- Rubric for each student
- colored pencils or markers

Name \_\_\_\_\_

### Native American Design Grade Sheet

1. Create your design using a Geometry drawing utility such as Geometer's Sketchpad, GeoGebra, or Cabri.
2. Print your design. (5 points) \_\_\_\_\_
3. Include a reflection (blue), translation (red) and rotation (yellow) in your design. (5 points each, total 15 points) \_\_\_\_\_
4. Your design must have at least one line of symmetry (green). (5 points)  
\_\_\_\_\_
5. Your design must have at least one example of rotational symmetry (purple). (5 points)  
\_\_\_\_\_
6. Does your design resemble any of the patterns seen in the PowerPoint of Native American Designs? If so, which tribal group does it most closely associate with? If not, explain how it is different. (15 points) \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
7. Which images, if any featured in the Native American Designs PowerPoint, represent designs from Montana tribes? You may use <http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>, if needed. (5 points) \_\_\_\_\_  
\_\_\_\_\_
8. You will also receive up to 5 points for design originality. (5 points)  
\_\_\_\_\_
9. Score out of 55 total points possible \_\_\_\_\_



# Native American Designs

- Many Native American designs have geometric properties.
- Planar Transformations
  - Pre-image-the original figure prior to a transformation occurring.
  - Image-the new figure produced after a transformation has occurred.
  - Reflection-a figure is *flipped* over a line of reflection, creating a mirror image of the figure.
  - Rotation-a figure is *turned* or rotated through a given angle and in a given direction about a fixed point called the center of rotation.
  - Translation-a *slide*, each point of a figure is moved the same distance in the same direction.

# Native American Designs

## ■ Symmetry

- ❑ **Line of Symmetry**-a figure has line symmetry if it can be divided by a line into two parts that are mirror images of each other.
- ❑ **Rotational Symmetry**-a figure has rotational symmetry if a turn of the smallest possible degree between 0 and 180 degrees produces an image that maps back onto the original figure.

---

# Native American Designs

- Look at the following Native American Designs.
  - ❑ Could the design have been created by a reflection? A translation? A rotation?
  - ❑ Does the design have a line or lines of symmetry?
  - ❑ Does the design have rotational symmetry?
- Images found at:

Morning Star Gallery

513 Canyon Rd.

Santa Fe, NM 87501

[indian@morningstargallery.com](mailto:indian@morningstargallery.com)

# Native American Designs

## Insert Description Including Tribe of Origin

Insert Image Here



Description, Tribe of Origin  
Circa XXXX

# Native American Designs

- **You will create your own design.**
  - ❑ Create your design using a Geometry drawing utility such as Geometer's Sketchpad, GeoGebra or Cabri.
  - ❑ Print your design. (5 points)
  - ❑ Include a reflection (blue), translation (red) and rotation (yellow) in your design. (5 points each, total 15 points)
  - ❑ Your design must have at least one line of symmetry (green). (5 points)
  - ❑ Your design must have at least one example of rotational symmetry (purple). (5 points)
  - ❑ You will also receive up to 5 points for design originality. (5 points)
  - ❑ Score out of 35 total points possible



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# Mathematics Model Teaching Unit

## Ko'ko'hasenestôtse

"The Art of Clicking Things Together"

Created by: Lisa Scott

Grade 8 – Approximate Duration: 110 minutes

### Stage 1 Desired Results

#### Established Goals:

**Data Analysis Mathematics Content Standard 2:** A student, applying reasoning and problem solving, will use data representation and analysis, simulations, probability, statistics, and statistical methods to evaluate information and make informed decisions within a variety of relevant cultural contexts, including those of Montana American Indians.

- **2.1 Representing and Comparing Data:** Collect data from a variety of contexts (e.g., science, history, and culture, including Montana American Indians). Organize and represent data in box plots, scatter plots, histograms, and circle graphs using technology when appropriate.
- **2.2 Evaluating Data and Making Conjectures:** Interpret, analyze, and evaluate data using mean, median, range, and quartiles to identify trends and make decisions and predictions about data within scientific and cultural contexts, including those of Montana American Indians.
- **2.3 Finding Probability and Predicting:** Create sample spaces and simulations from events found in different cultures, including those of Montana American Indians, determine experimental and theoretical probabilities, and use probability to make predictions.

**IEFA: Essential Understanding 3:** The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs.

Additionally, each tribe has its own oral histories, which are as valid as written histories. These histories pre-date the “discovery” of North America.

#### Understandings:

*Students will understand...*

- how to play Ko'ko'hasenestôtse, a game from the Cheyenne people.
- how to use experimental and theoretical probabilities to make predictions.
- how a large number of trials in an experiment can predict the theoretical probability of the event.

#### Essential Questions:

- How is the scoring of Ko'ko'hasenestôtse determined?
- How is the experimental probability determined?
- How is the theoretical probability determined?
- How is the Law of Large Numbers used to predict the theoretical probability?

*Mathematics Grade 8 – Ko'ko'hasenestôtse The Art of Clicking Things Together (continued)*

*Students will be able to...*

- understand and play the game of Ko'ko'hasenestôtse.
- collect and organize data into a table while simulating the game.
- determine the experimental probability of each throw.
- determine the sample space of the game (by listing or tree diagramming).
- use the Law of Large Numbers to predict the theoretical probability.
- determine the theoretical probability of each throw.
- use the probabilities to make predictions for future throws in the game of Ko'ko'hasenestôtse.

*Students will know...*

- the definitions of trial, experimental probability, sample space, Law of Large Numbers and theoretical probability.

**Stage 2 Assessment Evidence**

**Performance Tasks:** Worksheet with data collection, sample space, and probabilities. Worksheet with questions answered and turned in.

**Other Evidence:** Observation of game and data collection. Participation in class discussions. Individual questioning of students.

**Stage 3 Learning Plan**

**Learning Activities:**

**1. State the “Understandings” for the lesson**

**2. Warm Up Activity**

Put the following on the board and ask students to complete.

- Write 3 out of 4 as a ratio. ( $\frac{3}{4}$ , 3 to 4, or 3:4)
- What does probability mean? (How likely it is that an event will happen. The ratio of the number of favorably outcomes out of the total number of possible outcomes.)
- What is a tree diagram? Give an example. (A branching diagram to show all possible outcomes.)

**3. Handout, read and discuss the overview of the Northern Cheyenne Reservation**

**4. Inform the students that they will be playing Ko'Ko'hasenestôtse** (The Art of Clicking Things Together), which is a game from the Cheyenne people. Explain that it is sometimes referred to as Monshimout, but according to Dr. Richard Little Bear, President of Chief Dull Knife College, this name is an anglicized version of , Móheněšemahtôtse?, a Northern Cheyenne word meaning “card game”. The Ko'ko'hasenestôtse game, referred to as "Monshimout", can be found at:

(<http://www.manataka.org/page103.html#Cheyenne%20Basket%20Game>). Distribute directions using the desired format.

**5. Handout the Ko'ko'hasenestôtse Game Worksheet found at the end of this lesson.**

- Assign student partners
- Ask students to complete #1 on worksheet
- Handout the materials to play the game
- Have partners play the game as many time as they can in 20 minutes and record data in Table 1 & 2

*Mathematics Grade 8 – Ko'ko'hasenestôtse The Art of Clicking Things Together (continued)*

- Ask students to complete #2 through #5 on worksheet
- 6. Have a class discussion about #2 through #5**
- 7. Discuss how to determine the sample space and possible methods for listing all possible outcomes (list or tree diagramming)**
- 8. Ask students to complete #6 on the worksheet**
- 9. Discuss the number of outcomes in the sample space as a class (32 outcomes)**
- 10. Compile the class data by putting three columns on the board or in a projected spreadsheet**
  - The first column would be a copy of the first column of Table 1
  - Ask pairs to report how many tallies they had for each throw
  - Record the data for each pair and then find the total for the whole class for each throw
  - Write the total in the second column
  - In the third column write the experimental class probability
  - Ask students to copy these into Table 2

Throw	Total Number of Class Tallies	Class Experimental Probability
2B, 2P, 1C...		

- 11. Ask students to complete Table 2 and discuss the theoretical probabilities and how they were determined**
- 12. Answer questions #8 - #10**
- 13. Discuss the answers to #8 - #10 as a class and discuss the definition of the law of large numbers**
- 14. Ask students to complete #11 and #12 with their partner**
- 15. Turn in worksheet**

**Summary:** You have learned about the Cheyenne Basket Game called Ko'ko'hasenestôtse and a little bit of history on the people who live in Montana. You should now have a good understanding of experimental and theoretical probabilities and the law of large numbers.

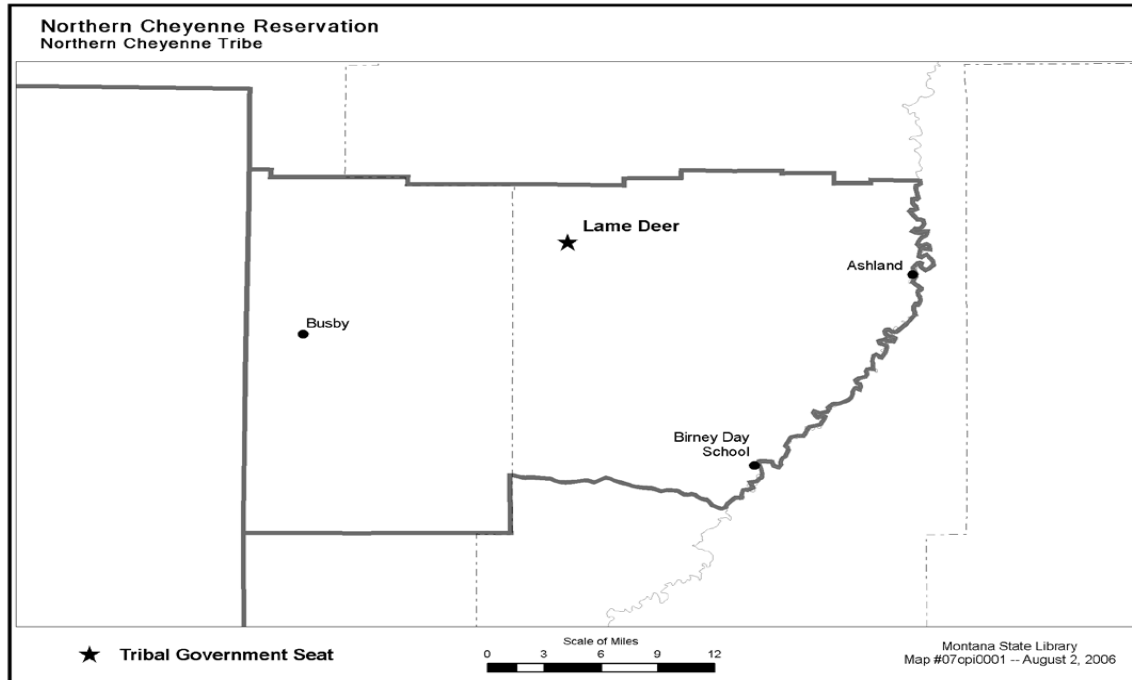
**Materials/Resources Needed:**

- Copies of the overview of the Northern Cheyenne Reservation for each student found at end of lesson or (<http://www.opi.mt.gov/indianed/studentbackground.html>).
- Copies of “Ko'ko'hasenestôtse Game” for each student. According to Dr. Richard Little Bear, President of Chief Dull Knife College, this Web site refers to this game as “Monshimout,” which is an anglicized version of, Móhenšemahtótse?, a Northern Cheyenne word meaning “card game.”
- (<http://www.manataka.org/page103.html#Cheyenne%20Basket%20Game>)
- “Ko'ko'hasenestôtse Game Worksheet” for each student, found at end of lesson.
- A “basket” (Dixie cup or small box) for each pair of students
- Five two colored chips (two that have “bear paw” markings on one side and three that have “cross markings” on one side and the opposite sides are blank) for each pair of students
- Eight “sticks” (straws or toothpicks) for each player



## Northern Cheyenne Reservation

### NORTHERN CHEYENNE TRIBE



### Location

The Northern Cheyenne Reservation, situated in southeastern Montana, lies within the counties of Big Horn and Rosebud. The Crow Reservation borders it on the west. The reservation consists of open ponderosa-pine plateau and valley country with an annual rainfall of approximately 16 inches. The topography ranges from about 4,800 feet to a low of a little less than 3,000 feet. The reservation headquarters and the center for business activities and population are in Lamé Deer. The reservation itself is divided into five districts; Busby, Lamé Deer, Ashland, Birney, and Muddy.

### Population

Total number of enrolled tribal members Approximately 7,374

Even though there are over 7,000 enrolled members, about 4,199 members live on the reservation scattered through the five district communities.

There is also a relatively small population of non-Indians and other tribal members living on the reservation.

### **Land Status**

Total acres within the reservation's boundary 444,774.50 acres

Individually allotted lands 113,277.70 acres

Tribally owned lands 326,546.81 acres

Fee title or state lands 4,827.70 acres

Non-Indians own about 30 percent of the fee or state lands on the Northern Cheyenne Reservation. The Tribal Council has selected a Land Acquisitions Committee whose primary policy is directed to the purchase of land into Tribal ownership. The Committee thus assures that Indian land is retained in Indian ownership.

### **Historical Background**

The Cheyenne Indians are part of a linguistic group of the Algonquian language stock. Originally, it is believed that the ancestors of the Cheyenne lived south of the Hudson Bay and James Bay areas and slowly moved west into what is now northwestern Minnesota where the Red River forms a border between Minnesota and the Dakotas. During the late 1600s, they settled among the Tribes of the upper Missouri River and began farming rather than subsisting as small game hunters and fishermen. During the early 1700s, they were still primarily farmers growing corn, but they also hunted buffalo. The Cheyenne acquired the horse around 1750, and made the transition from a horticultural existence to a horse culture within a matter of several generations. Hunting buffalo became a way of life as they migrated as far south as New Mexico and Texas. The Cheyenne participated in the treaty making in 1825 near what is now Fort Pierre, South Dakota. A few years later, the larger part of the tribe (now the southern Cheyenne) moved southward and occupied much of the Arkansas River in Colorado and Kansas. The remainder continued to inhabit the plains from the headwaters of the North Platte up on to the Yellowstone River in Montana. The division of the tribe was recognized by the Fort Laramie Treaty of 1851. In the Battle of the Little Bighorn in 1876, the Northern Cheyenne joined the Sioux in what the Cheyenne call "where Long Hair was wiped away forever." Cheyenne oral history recalls a time when George A. Custer smoked a Cheyenne pipe and vowed never to fight the Cheyenne again. The ashes from the pipe dropped on his boot and scattered on the ground. These ashes were wiped away signaling Custer's commitment never to fight the Cheyenne again. Although the Cheyenne won the battle it was the beginning of the end for them as they were exiled to Indian Territory in Oklahoma to be colonized with the Southern Cheyenne. A small band escaped in a desperate effort led by Chief Dull Knife (Morning Star) and Chief Littlewolf. These two chiefs, in one of the most heroic episodes of western history, bravely fought against overwhelming odds, leading a small band of men, women, and children back to their homelands. The Northern Cheyenne call themselves "the Morning Star people." The name is taken and used in respect of Chief Dull Knife who was also known as Morning Star. Chief Littlewolf and Chief Dull Knife are buried side by side in the Lane Deer cemetery. By Executive Order of November 26, 1884, a tract of country east of the Crow Reservation was set apart as a reservation for the Northern Cheyenne. The reservation was expanded by another Executive Order in 1900 to its present boundaries.

(2007. *Montana Indians: Their History and Location*. Helena, MT: Office of Public Instruction.

<http://www.opi.mt.gov/indianed/studentbackground.html>)

## Ko'ko'hasenestôtse Game Worksheet

Name \_\_\_\_\_

1. Predict how many times you think you would need to throw the stones before you would see the winning throw of 2 bears and 3 crosses. \_\_\_\_\_ (Write your prediction as a ratio of 1 success or win to the number of throws.)

2. Play the game of Ko'ko'hasenestôtse with a partner. Tally each trial (the number of times you obtain “sticks” from your partner) for the following throws. **B = blank, P = bear paw and C = cross**. Play the game one, two or three times. If you play more than one game, continue to tally in the same table below and record how many throws it took to win each game. Determine the experimental probability for each throw using all the data from each game combined. (Experimental probability is the number of successes out of the number of trials).

Throw	# of Sticks	Tally	Experimental Probability
2 blanks, 2 bears, 1cross	0		
4 blanks, 1 bear, 0 crosses	0		
5 blanks, 0 bears, 0 crosses	1		
3 blanks, 2 bears, 0 crosses	1		
1 blank, 2 bears, 2 crosses	1		
2 blanks, 0 bears, 3 crosses	3		
0 blanks, 2 bears, 3 crosses	8		
Other 0			

Table 1

Game Number	Number of throws to win the game
1	
2	
3	

Table 2

3. Which throw(s) from Table 1 seemed the most likely to happen? Explain.

4. Which throw(s) from Table 1 seemed the least likely to happen? Explain.

5. Using Table 2, how many times did you have to throw until someone won the game? Do you think this would be the same or close to the same if you were to play again? Why or why not?

6. Determine the sample space (every possible way the five stones may land) by listing or tree diagramming. Show all of your work.

7. Copy the experimental probabilities from Table 1 in step 2. Your teacher will help you compile the class data. Determine the experimental probability for each throw using the combined class data. Determine the theoretical probability of each throw and write those in Table 3. (Theoretical probability is the number of successes out of the total number of outcomes in the sample space.)

Throw	# of Sticks	Game Experimental Probability	Class Experimental Probability	Theoretical Probability
2 blanks, 2 bears, 1 cross	0			
4 blanks, 1 bear, 0 crosses	0			
5 blanks, 0 bears, 0 crosses	1			
3 blanks, 2 bears, 0 crosses	1			
1 blank, 2 bears, 2 crosses	1			
2 blanks, 0 bears, 3 crosses	3			
0 blanks, 2 bears, 3 crosses	8			
Other 0				

Table 3

8. How do the game experimental probability and the class experimental probability compare? Which do you think more accurately reflects the probability of each throw if you were to play multiple times? Explain why.

9. How do the experimental probabilities and theoretical probability compare? Should they be close in value? Why or why not? Which probability is a more accurate prediction of this game if you were to play it multiple times? Why?

10. Which experimental probability is closest to the theoretical probability? If you were to continue to throw the stones, how many times do you think you would need to throw and accurately predict the theoretical probability? Why do you think this? (Law of large numbers states that if you repeat a random experiment a large number of times, your outcomes should on average equal the theoretical average.)

11. Compare the theoretical probability to the prediction you made in question 1. How close was your prediction? Use the theoretical probability to predict how many times you should expect to get 2 bears and 3 crosses if you were to throw the stones 500 times? Show your work.

12. Do you think the Cheyenne people used the theoretical probability of each throw to determine the number of sticks you would obtain? Why or why not? Use the probabilities in your explanation.

# Ko'ko'hasenestôtse Game

## Worksheet (Answer Key)

Name \_\_\_\_\_

1. Predict how many times you think you would need to throw the stones before you would see the winning throw of 2 bears and 3 crosses. Any answer is acceptable (approx. 1/32) (Write your prediction as a ratio of 1 success or win to the number of throws.)

2. Play the game of Ko'ko'hasenestôtse with a partner. Tally the number of times you obtain “sticks” from your partner for the following throws. **B = blank, P = bear paw and C = cross** . Play the game one, two or three times. If you play more than one game, continue to tally in the same table below and record how many throws it took to win each game. Determine the experimental probability for each throw using all the data from each game combined. (Experimental probability is the number of successes out of the number of trials). *Sample Game*

Throw	# of Sticks	Tally	Experimental Probability
2 blanks, 2 bears, 1cross	0	III	$\frac{4}{25}$
4 blanks, 1 bear, 0 crosses	0	II	$\frac{2}{25}$
5 blanks, 0 bears, 0 crosses	1		$\frac{0}{25}$
3 blanks, 2 bears, 0 crosses	1	I	$\frac{1}{25}$
1 blank, 2 bears, 2 crosses	1	II	$\frac{2}{25}$
2 blanks, 0 bears, 3 crosses	3	I	$\frac{1}{25}$
0 blanks, 2 bears, 3 crosses	8		$\frac{0}{25}$
Other	0	IIIIIIIIII	$\frac{15}{25}$

Table 1

Game Number	Number of throws to win the game
1	29
2	32
3	35

Table 2

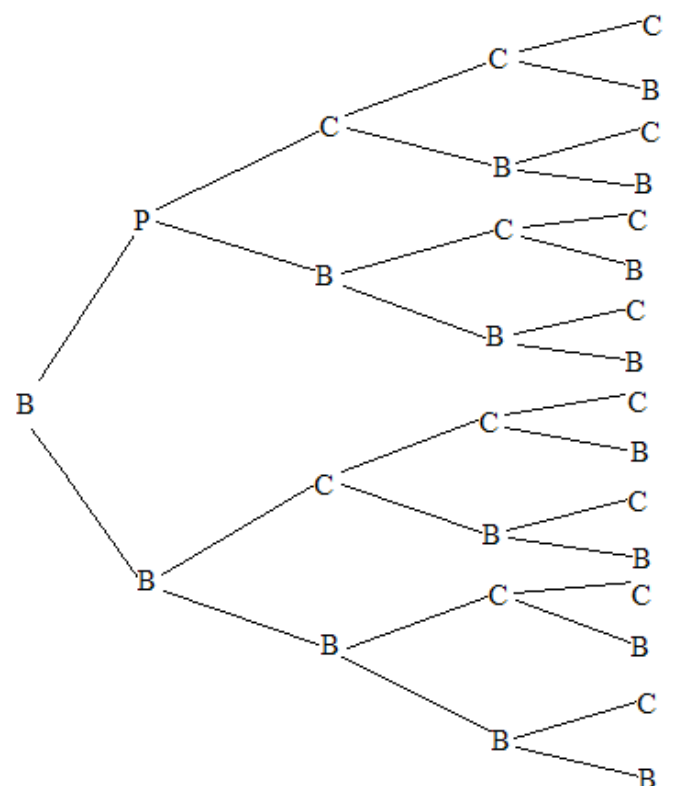
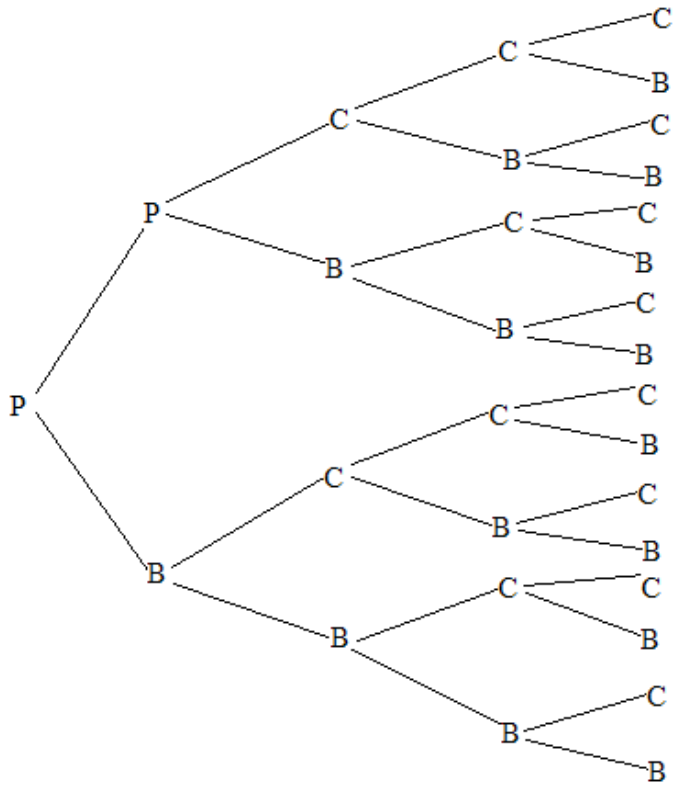
3. Which throw(s) from Table 1 seemed the most likely to happen? Explain. *(Sample response) The “other” and 2B, 2P, 1C. The “other” had 15 tallies out of 25. These two seemed most likely. The 2B, 2P, 1C occurred 4 out of 25. It was the second most likely to occur.*

4. Which throw(s) from Table 1 seemed the least likely to happen? Explain. *(Sample response) The 5B and 2P, 3C happened zero times each. 3P, 2C and 2B, 3C only happened one time each. None of these seem very likely to occur.*

5. Using Table 2, how many times did you have to throw until someone won the game? Do you think this would be the same or close to the same if you were to play again? Why or why not? *(Sample response) I threw 25 times and only collected 6 of my opponent’s sticks. So, I know I would have to throw more than 25 to win. I think the results would be similar. The chances of each throw shouldn’t change. So, the results should be similar.*

6. Determine the sample space (every possible way the five stones may land) by listing or tree diagramming. Show all of your work.





*Sample Space (32 outcomes)*

<i>PPCCC</i>	<i>PPCCB</i>	<i>PPCBC</i>	<i>PPCBB</i>	<i>PPBCC</i>	<i>PPBCB</i>	<i>PPBBC</i>
<i>PPBBB</i>	<i>PBCCC</i>	<i>PBCCB</i>	<i>PBCBC</i>	<i>PBCBB</i>	<i>PBBCC</i>	<i>PBBCB</i>
<i>PBBBC</i>	<i>PBBBB</i>	<i>BPCCC</i>	<i>BPCCB</i>	<i>BPCBC</i>	<i>BPCBB</i>	<i>BPBCC</i>
<i>BPBCB</i>	<i>BPBBC</i>	<i>BPBBB</i>	<i>BBCCC</i>	<i>BBCCB</i>	<i>BBCBC</i>	<i>BBCBB</i>
<i>BBBCC</i>	<i>BBBCB</i>	<i>BBBBC</i>	<i>BBBBB</i>			

7. Copy the experimental probabilities from Table 1 in step 2. Your teacher will help you compile the class data. Determine the experimental probability for each throw using the combined class data. Determine the theoretical probability of each throw and write those in Table 3. (Theoretical probability is the number of successes out of the total number of outcomes in the sample space.)

Throw	# of Sticks	Game Experimental Probability	Class Experimental Probability	Theoretical Probability
2 blanks, 2 bears, 1 cross	0	$\frac{4}{25}$	$\frac{3}{32}$	$\frac{3}{32}$
4 blanks, 1 bear, 0 crosses	0	$\frac{2}{25}$	$\frac{2}{32}$	$\frac{2}{32}$
5 blanks, 0 bears, 0 crosses	1	$\frac{0}{25}$	$\frac{1}{32}$	$\frac{1}{32}$
3 blanks, 2 bears, 0 crosses	1	$\frac{1}{25}$	$\frac{1}{32}$	$\frac{1}{32}$
1 blank, 2 bears, 2 crosses	1	$\frac{2}{25}$	$\frac{3}{32}$	$\frac{3}{32}$
2 blanks, 0 bears, 3 crosses	3	$\frac{1}{25}$	$\frac{1}{32}$	$\frac{1}{32}$
0 blanks, 2 bears, 3 crosses	8	$\frac{0}{25}$	$\frac{1}{32}$	$\frac{1}{32}$
Other	0	$\frac{15}{25}$	$\frac{20}{32}$	$\frac{20}{32}$

Table 3

8. How do the game experimental probability and the class experimental probability compare? Which do you think more accurately reflects the probability of each throw if you were to play multiple times? Explain why. *(Sample response) The probabilities are close in value. I think they should be. They would probably get closer the more times I throw the stones. They are close because the chances of each throw should stay the same, as the tree diagram shows.*

9. How do the experimental probabilities and theoretical probability compare? Should they be close in value? Why or why not? Which probability is a more accurate prediction of this game if you were to play it multiple times? Why? *(Sample response) The class probability and the theoretical probability are somewhat close in value. I think they should be. They would probably get even closer in value the more times I throw the stones. I think the class probability begins to predict what should happen for anytime I play. The theoretical is what should happen. I think they should be close because the chances of each throw shouldn't change very much, as the tree diagram shows.*

10. Which experimental probability is closest to the theoretical probability? If you were to continue to throw the stones, how many times do you think you would need to throw and accurately predict the theoretical probability? Why do you think this? (Law of large numbers states that if you repeat a random experiment a large number of times, your outcomes should on average equal the theoretical average.) *(Sample response) The class probability is closer to the theoretical probability. I think the more times I throw the stones the closer the probability will become to the theoretical probability.*

11. Compare the theoretical probability to the prediction you made in question 1. How close was your prediction? Use the theoretical probability to predict how many times you should expect to get 2 bears and 3 crosses if you were to throw the stones 500 times? Show your work. *(Sample response) My prediction was fairly close. I would take the theoretical probability of  $\frac{1}{32}$  and multiply it to the 500 times I would throw the stones. So, I think I would get 2 bears and 3 crosses about 15 or 16 times in 500 throws.*

12. Do you think the Cheyenne people used the theoretical probability of each throw to determine the number of sticks you would obtain? Why or why not? Use the probabilities in your explanation.

*(Sample response) I don't think they took the probabilities into consideration when assigning the number of sticks you would take from your opponent. I think this because the theoretical probability of getting 2B, 2P, 1C is  $\frac{3}{32}$  and you don't get to take any sticks if you throw this. 1B, 2P, 2C is also  $\frac{3}{32}$  and you get to take 1 stick. 4B, 1P is  $\frac{2}{32}$  and you don't get to take any sticks. 5B and 3B, 2P are both  $\frac{1}{32}$  and you get 1 stick for each. But 2B, 3C is  $\frac{1}{32}$  and you get to take 3 sticks. 2P, 3C is  $\frac{1}{32}$  and you take all eight. It seems to me that the probabilities that are the same should have the same number of sticks taken and the combinations with the lowest probability should be worth the most sticks and the combinations with the higher probabilities should be worth the least number of sticks.*



Montana  
Office of Public Instruction  
Denise Juneau, State Superintendent

# Mathematics Model Teaching Unit

## Surface Area and Volume of Traditional American Indian Homes

Created by: Joe Meyer

Grade 8 – Approximate Duration: 100 minutes

### Stage 1 Desired Results

#### Established Goals:

**Geometric Reasoning Mathematics Content Standard 3:** A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant cultural contexts, including those of Montana American Indians.

- **3.1 Properties of Solids and Figures:** Define, classify and compare properties of solids and plane figures, including lines and angles.

**IEFA Essential Understanding 1:** There is great diversity among the 12 tribal Nations of Montana in their languages, cultures, histories and governments. Each Nation has a distinct and unique cultural heritage that contributes to modern Montana.

**IEFA Essential Understanding 2:** There is great diversity among individual American Indians as identity is developed, defined and redefined by entities, organizations and people. A continuum of Indian identity, unique to each individual, ranges from assimilated to traditional. There is no generic American Indian.

#### Understandings:

*Students will understand...*

- that there are several different types of traditional American Indian houses.
- that traditional American Indian houses contain various plane and solid geometric figures.
- how to find surface area of various plane geometric figures.
- how to find volume of various solid geometric figures.

#### Essential Questions:

- What are some types of traditional American Indian houses?
- What plane and solid geometric shapes do some traditional American Indian houses contain?
- How do you calculate surface area and volume of traditional American Indian houses?
- How would knowing surface area and volume help a builder construct a building?

*Mathematics Grade 8 – Surface Area and Volume of Traditional American Indian Homes (continued)*

*Students will be able to...*

- identify a variety of geometric plane and solid figures.
- calculate area of geometric plane figures and surface area and volume of geometric solids.

*Students will know...*

- how to find area of various geometric plane figures.
- how to find surface area and volume of various geometric solid figures.

**Stage 2 Assessment Evidence**

**Performance Tasks:**

- Build a structure out of wooden dowels and rubber bands.
- Compare geometric plane and solid shapes to traditional American Indian houses.
- Identify a variety of geometric plane and solid figures.

**Other Evidence:**

- Teacher observations of student participation during all portions of the lesson.
- Student presentations.
- Peer assessment and input during small-group activities.
- Question responses of individual students.
- Homework assignment (found later in the lesson).

**Stage 3 Learning Plan**

**Learning Activities:**

**A. Open and “hook”:**

1. Use cards chosen out of a standard deck to allow for random grouping upon entry into the classroom by the students. For groups of two, use 2 of each card number. For groups of 3, use 3 of each card number. For instance, all the Aces will be in a group, all the Kings will be in a group, etc.
2. As students enter the room, have them draw a card out of the deck. Ask them to find the other student(s) that have the same number on their card(s) and sit at a table or group of desks.
3. State the “Understandings” for the lesson.
4. Show the kids some dowels and a bag of rubber bands. Explain that each group will get 16 dowels and a bag of rubber bands. Inform the kids they are going to make a structure out of these materials that has to be tall enough for a 4-foot tall person to stand in it.
5. Issue the materials to each group, and inform them they have 10 minutes to complete their structures and all in the pair/group must participate.

**B. Survey ideas for naming plane and solid figures:**

1. Ask students to go around the classroom. Inform them they are to record the names of every plane and solid figure they can find on the dowel structures, and whose structure it belongs to, on paper. Give them 3 minutes to do this.
2. Record the information the students wrote down on the board in 2 columns, one for plane figures and one for solid figures (sample responses may include: rectangles, triangles, squares, pentagons for planes; various pyramids or prisms for solids).
3. Discuss as a class.
4. Distribute the Student Area and Volume Handout to each student.

*Mathematics Grade 8 – Surface Area and Volume of Traditional American Indian Homes (continued)*

5. Ask the students to decide which of the shapes from the handout were created in their constructions. Have the students do this in a “Think, Pair, Share” format (think about it, share with the person they sat by, and be ready to share with the class).
6. Discuss as a class.

C. Familiarizing students with traditional American Indian housing:

1. Distribute a student handout, which you create, pertaining to different types of traditional American Indian housing, make overheads to display for the students, or project the webpage that can be found at <http://www.native-languages.org/houses.htm>.
2. Have each student decide which (if any) of the traditional American Indian housing their dowel structure resembles. Be sure to discuss the writings accompanying each house style.
3. Have each group present orally to the class the type of house they chose and explain why. If a group didn't come up with one, assign one that the other kids won't be utilizing. Each group should determine which geographic region of Native Americans most likely utilized this type of dwelling. In addition, groups should determine, which, if any, Montana tribal group(s) would have utilized this dwelling.

D. Calculate surface area and volume for the dowel houses chosen by each group:

1. In groups, have students sketch a picture of the traditional housing type that they chose. Tell them to use geometric plane and solid figures from the first student handout in their drawings. Have students complete all of the steps individually, utilizing their partner for assistance and comparisons.
2. Ask the students to label next to each shape that makes their house what it is (i.e.—circle, triangular prism, rectangle, cone, etc.)
3. Using the handout from part “C”, have the students label dimensions on their sketch to the nearest foot. The teacher may need to assist with estimations if actual dimensions are not provided.
4. Discuss how they could calculate how much material they might need to create a fully-covered house. Lead the discussion towards surface area.
5. Have the students calculate the surface area of their house, explaining their process and reasoning to their partner as they progress. One group member can be a “traveler” if the group is stuck to consult another group for assistance. Circulate about the room, and assist if needed.
6. Have the students share what they did to the class on the board. All group members must participate in the presentation.
7. Complete steps 4 through 7 above for the volume of their house.

E. Assign Surface Area and Volume of Traditional American Indian Homes

1. How would knowing surface area of a traditional home help the builder? How about builders of modern day homes? **Possible answer: to know approximately how much material would be needed to build.**
2. How would knowing volume of a traditional home help the builder? How about builders of modern day homes? **Possible answer: to know approximately how much space would there be inside.**
3. Find two different solid shapes in real life. Select items that can be found on our formula sheet.
4. Draw a picture of each and explain what the shapes represent.
5. Measure the dimensions, and label them on your picture.
6. Calculate the surface area and volume of the two shapes.
7. Be ready to present the results of your homework in class tomorrow.

*Mathematics Grade 8 – Surface Area and Volume of Traditional American Indian Homes (continued)*

**Materials/Resources Needed:**

- A deck of standard playing cards.
- Enough wooden dowels and bags of rubber bands for each group. Each group will need sixteen 3-foot wooden dowels and a bag of rubber bands. If access or money is an issue, plastic straws could be substituted for wooden dowels, but lesson steps would have to be modified.
- Copies of the student handouts for each student (furnished at the end of the lesson). You may also choose to make overhead transparencies of the handouts for discussion purposes.

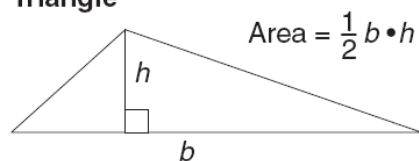
**References:**

- Montana Office of Public Instruction CRT Math Reference Sheet Grade 8
- <http://www.native-languages.org/houses.htm>

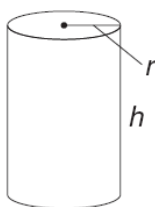
Name \_\_\_\_\_

Student Area and Volume Handout

**Triangle**

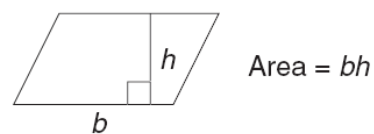


**Cylinder**

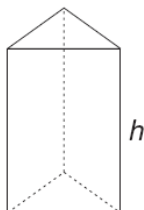


Volume =  $\pi r^2 h$   
Surface Area =  $2\pi r^2 + 2\pi rh$

**Parallelogram**

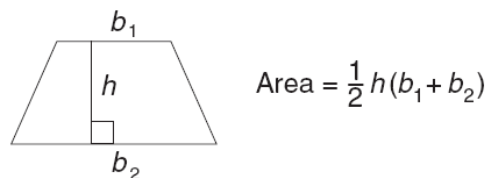


**Prism**

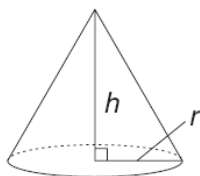


Volume =  $Bh$   
where  $B$  = area of base

**Trapezoid**

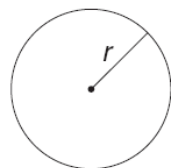


**Cone**



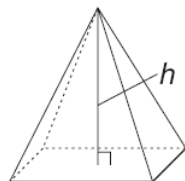
Volume =  $\frac{1}{3} \pi r^2 h$

**Circle**



Area =  $\pi r^2$   
Circumference =  $2\pi r$

**Pyramid**



Volume =  $\frac{1}{3} Bh$   
where  $B$  = area of base



Montana Office of Public Instruction CRT Math Reference Sheet Grade 8

**Assignment for students:**

Name \_\_\_\_\_

Surface Area and Volume of Traditional American Indian Homes

1. How would knowing surface area of a traditional home help the builder? How about builders of modern day homes?
2. How would knowing volume of a traditional home help the builder? How about builders of modern day homes?
3. Find two different solid shapes in real life. Be sure the items have shapes that can be found on our formula sheet.
4. Draw a picture of each and explain what the shapes represent.
5. Estimate or measure their dimensions, and label them on your picture.
6. Calculate the surface area and volume of the two shapes.

Be ready to present the results of your homework in class tomorrow in groups of 4.



Montana  
Office of Public Instruction  
Denise Juneau, State Superintendent

# Mathematics Model Teaching Unit

## Pow Wow Trails

Created by: Linda Engebretson

Grade 9 - Approximate Duration: 110 minutes

### Stage 1 Desired Results

#### Established Goals:

#### Algebraic and Functional Reasoning Mathematics Content Standard 4

A student, applying reasoning and problem solving, will use algebraic concepts and procedures to understand processes involving number, operation, and variables and will use procedures and function concepts to model the quantitative and functional relationships that describe change within a variety of relevant cultural contexts, including those of Montana American Indians.

- **4.3 Solving Systems of Equations and Inequalities:** Solve a variety of equations, inequalities and systems of equations and inequalities, justify the solution process, and interpret the solution in context.
- **4.5 Analyzing and Conjecturing with Models:** Given data or a problem situation, select and use an appropriate function model to analyze results or make a prediction with and without technology using cultural contexts, including those of Montana American Indians.

**IEFA Essential Understanding 2:** There is great diversity among individual American Indians as identity is developed, defined and redefined by entities, organizations and people. A continuum of Indian identity, unique to each individual, ranges from assimilated to traditional. There is no generic American Indian.

**IEFA Essential Understanding 3:** The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs.

Additionally, each tribe has its own oral histories, which are as valid as written histories. These histories pre-date the “discovery” of North America.

#### Understandings:

- Pow wows are an important part of some Native American lifestyles.
- Algebra is used to solve real-world problems.
- Basic operations are used to solve algebraic expressions.
- Estimates in measurement.
- Problem solving skills.
- Proportions are two equal ratios.

#### Essential Questions:

- What is the history of pow wows in Native American culture?
- Why is estimating important?
- How will the budget be determined?
- What is the importance of the scale factor of a map?
- How is a ratio applied in calculating a real-world problem?

*Mathematics Grade 9 – Pow Wow Trails (continued)*

*Students will be able to...*

- read a table with dates and time.
- calculate the mileage of a vehicle.
- estimate the cost.
- calculate amount of money needed.
- determine the distance using the scale factor or scale from a map.
- apply fraction and percent operations.

*Students will know...*

- pow wows are still in existence and that families use them as a time of gathering and sharing with other families and friends.
- the timeline for their travels.
- how to find the total distance using a scale factor or a scale from the map.
- how to develop a budget.
- how to apply basic operations in doing the calculations of the cost and percents.

**Stage 2 Assessment Evidence**

**Performance Tasks:**

- Calculate the total miles by using the scale factor or bar scale on a map
- Calculate the total cost of gas using the mileage, distance, and miles per gallon of the vehicle.
- Estimate the cost of the Pow Wow.
- Calculate if there will be sufficient money for the Pow Wow trail they have created.
- Apply percentages in estimating the cost for the Pow Wow trail.

**Other Evidence**

- Journal of each Pow Wow describing the categories of dancers, drum groups, and games
- The budget that was developed
- Quiz over using cross products
- Quiz over finding percent

**Stage 3 Learning Plan**

**Teacher notes:**

“A Pow Wow is a gathering where Native American dancing, singing and celebration take place. There are several different powwows that take place throughout the country.... Some reports say the word pow wow has its origin from the Pawnee word pa-wa, meaning ‘to eat.’ Other sources say the word is of Algonquin origin and was originally pronounced pauau, which indicates a gathering of people for purpose of celebration or important event. In any case, it is a special time for people to gather and celebrate, meet old friends and create new friendships.” *Your Guide to Understanding and Enjoying Pow Wows*. To learn more about Pow Wows look at the following Web site; <http://www.opi.mt.gov/pdf/IndianEd/Resources/PowWows.pdf>.

General rules to Pow Wow rules to follow;

- Stand during grand entry
- Do not enter the dance arbor once it has been blessed
- Never call the dancer’s regalia a “costume” (outfit is acceptable)
- Respect everyone; non-Native, Native and especially the elders
- Have fun

### *Mathematics Grade 9 – Pow Wow Trails (continued)*

This activity could be used at the beginning of the school year. It is a way to get their minds moving again by reviewing fractions, decimals, estimations, and calculations applying algebraic processes. This is a non-threatening way to get students involved and it is something the student has control over in the planning.

#### **Procedure:**

1. Introduce the history of powwows and other background information about pow wows. Explain how some American Indian families travel from pow wow to pow wow on what is called the “pow wow circuit” in Montana, using the time to camp and compete (get background information from Your Guide to Understanding and Enjoying Pow Wows which can be found at [www.opi.mt.gov/IndianEd](http://www.opi.mt.gov/IndianEd)).
2. Give every student a map of Montana and a list of the Pow Wows in the state of Montana. The Pow Wows for the years 2007 and part of 2008 are attached.
3. Tell students they get \$1200.00 to create a budget that will take them to four different Pow Wows.
4. Students will budget the money for the following expenses:
  - Gasoline
  - Motel if they are not staying with family or friends
  - Meals (two per day)
  - Participation of games
  - Purchase of gifts from the vendors
  - Snacks and drinks while at the Pow Wow
  - Emergency: add 10% to the total of cost of gasoline, meals, motels, gift purchase, and snacks
5. After the students have been given their maps, the Pow Wow schedule and the total amount of money available to spend, have them find their trail.
6. Once students have determined their route or trail, as a class, find the scale on the map.
7. Discuss as a class: What is a map scale? (Answer: a single unit of distance on the map that is equivalent to the distance on the ground) There are two basic scales for maps:
  - Ratio: this is a scale represented or expressed as a fraction or a ratio. For example, 1:25 (ratio) represents one unit on the map for 25 units on the ground or 1 inch represents 25 miles on the ground.
  - A graphical or bar scale is ruler with ground distances added. The distance on the map is marked on the edge of a sheet of paper, then placed on the bar scale of the map and the distance read.
8. Students will find the total distance traveled on their Pow Wow trail. Find the total distance traveled by either applying the scale factor (or ratio) or using the bar scale. Here is an example of applying a scale factor (ratio) in finding the distance:

$$\text{Total distance} = (\text{scale factor}) \times (\text{total units measured from the map})$$

*Mathematics Grade 9 – Pow Wow Trails (continued)*

If a bar scale is being used, have students take the edge of a piece of paper and mark the edge, finding the distance.

9. Once all the students have found the distance, find the cost of gasoline. Before the cost of gasoline can be calculated, students must have an idea of the mileage of their vehicle. If they do not know this, use an average value of 20 miles per gallon of gasoline. Have students write down the mileage or use the average value of 20 miles per gallon.

10. Discuss as a class how the mileage of a vehicle can be calculated. (Answer:  $\frac{\text{miles}}{\text{gallons}} = \text{mileage}$ )

Also discuss with the class if units are important in doing the calculations? (Answer: Units help in setting up the problem correctly and help in determining what is being calculated.)

11. Discuss with the class how the amount of gasoline for the Pow Wow trail can be calculated. Give students time to experiment with this to find the solution. Show all students (by writing on the board) one way to set this problem up:

$$\frac{20 \text{ miles}}{1 \text{ gallon}} = \frac{\text{total miles from Pow Wow trail}}{\# \text{ of gallons of gasoline used}}$$

11. Discuss with the class how to solve this problem. Suggest that a cross product is an easy way to solve it. Walk through solving this by completing it on the board with the whole class, showing how the units will divide out leaving only gallons, which is exactly what we are looking to find. Remind students how a cross product works: multiply the means by the extremes and set them equal to each other.

- In the example above the **means** would be: 1 gallon • total miles  
and the **extremes** would be: # of gallons • 20 miles.
- It would look like this: **1 gallon • total miles = # of gallons • 20 miles.**

The number of miles will be known and we will divide by the 20 miles. The unknown is the number of gallons. Once students have found the number of gallons of gas to be used, have them write this number down below the mileage. Make sure students label their answers.

12. Once the amount of gasoline is calculated, the cost can be found for the gasoline. Discuss with the class how this could be found. **Give students time to think about this problem. Allow them to experiment again and take suggestions as to how to set it up. After giving students time to think and experiment with this, coach them into setting it up like this: # of gallons • cost per gallon = cost for gasoline.** Have students write this calculation down below the number of gallons expected to be used. Remember to label the answers.

*Mathematics Grade 9 – Pow Wow Trails (continued)*

13. Discuss as a class what other expenses need to be calculated while on their Pow Wow trail. (Students should consider meals, motel rooms, cost of the food, games and vendors while at the Pow Wow and have some money set aside for emergencies.)
14. Take suggestions from students how the cost of meals could be calculated.  
(Example:  $\text{Cost per meal} \cdot \text{number of meals per day} \cdot \text{number of days on the Pow Wow trail} = \text{cost of meals}$ )
15. Have students determine the cost of their meals. Write this cost down below the cost of the gasoline.
16. Next have students determine if they will be staying with family or have to find a motel room. Maybe they have a camper or trailer they can stay in. This will be an individual determination in the cost. However, the cost of a motel room during peak season would be approximately \$60.00 - \$70.00 per night. The price on these motels is a conservative estimate. Remind students they will be visiting four Pow Wows.
17. Have students write this cost of the motels, or the cost for a camper or trailer spot below the cost of meals.
18. Discuss with the class how much money will be needed to buy presents from the vendors. Discuss buying snacks while at the Pow Wow. (Snacks such as homemade lemonade, fry bread and Indian tacos are some of the foods usually sold at a Pow Wow.) Have students determine how much they will spend at each Pow Wow. Each student should write this amount below the cost for the motel or the camping spot.
19. Find the total for the Pow Wow trail by adding the cost of gasoline + cost of meals + cost of motels + cost of presents + cost of snacks = total cost.
20. Finally, discuss with the class how they could budget for emergencies. Suggest to the class about adding 10% onto the total cost. After finding the total cost (cost of gasoline + cost of meals + cost of motels + cost of presents + cost of snacks), have students find 10% of this amount and add it to the total cost.
21. Review with the class how to find the percent of a number. Remind students that 10% is 10/100 or 1/10 or 0.1. Have students write 10% as a fraction or as a decimal.
22. Have each student multiply the total cost of their Pow Wow by 0.1 or 1/10. Once this amount is found, add this to the total that was calculated in step 19. Now students have a cushion for emergencies.
23. Once students have calculated the cost and added 10% onto to their total, they need to look at their budget and make sure that the \$1200.00 will cover their Pow Wow trail.
24. What are some solutions if students go over their \$1200.00 budget? Have the class develop solutions for this problem.

*Mathematics Grade 9 – Pow Wow Trails (continued)*

**Resources:**

<http://www.opi.mt.gov/pdf/IndianEd/Resources/PowWows.pdf>

<http://www.visitmt.com/categories/ListCalendar.asp?Title=Pow+Wows+in+Montana&SiteTypes=Pow+Wow&SiteID=1>

**Materials Needed:**

Maps of Montana or other states (available from Montana Travel, Chamber of Commerce, or travel guides in communities)

Highlighters

Journals

List of Pow Wows in the state of Montana (see attached)

Calculator

## Pow Wows in the state of Montana

### Fort Kipp Celebration

**Poplar 406-786-3431**

Fort Kipp Celebration is a Pow Wow, which is a celebration of native culture and traditions through dancing, food, crafts, and fellowship with one another. All members of the General Public are welcome to participate or be a spectator.

### Arlee 4th of July Celebration

**Arlee 406-675-2700 ext 1222**

Beginning its second century the Arlee 4th of July Celebration will begin on Thursday which is designated 'Camp Day'. Many participants will begin erecting encampments for the coming weekend. The evening is set-aside for Memorial Ceremonies, which acknowledge the tribal members who passed away.

### 4th of July Pow Wow

**Lame Deer 406-477-6284** The premier event of the Northern Cheyenne is the annual Fourth of July Celebration, the largest wow-wow held on the reservation. Activities to observe and participate in include fun runs and health walks. The Princess Contest (all princesses welcomed), Indian dancing contests in all categories

### Valley of The Chiefs Pow Wow and Rodeo

**Lodge Grass 406-638-2073** Valley of The Chiefs Pow Wow and Rodeo includes: rodeo, parade, Indian dancing, authentic costumes, teepee camping, and parade dance through the village (breaking camp) on the last day.

### North American Indian Days

**Browning 406-338-7276** North American Indian Days, an annual celebration and the largest and most impressive of Blackfeet Tribal events, hosts Native Americans from every region of the United States and Canada. Featured events include traditional drumming and dancing contests, the crowning of Miss Blackfeet, and a parade.

### Wahcinca Dakota Oyate Celebration

**Poplar 406-768-5186**

The Wahcinca Dakota Oyate Celebration is a Pow Wow, which is a celebration of native culture and traditions through dancing, food, crafts and fellowship with one another. All members of the general public are welcome to participate or be a spectator. \$125/day for food stands, \$75/day for craft stands.

### Standing Arrow Pow Wow

**Elmo 800-883-5344** This is an Indian social gathering featuring drumming, dancing, and traditional dress and food. Visitors are welcome and asked to respect the dance area, which is sacred. Join us at one of the Indian country's most publicized events, Pow Wows. Learn the how's and why's of Pow Wow dancing.

### Milk River Indian Days

**Fort Belknap 406-353-2205 ext 367** Milk River Indian Days highlights spirited and colorfully clad dancers. The Pow Wow features Native American dancers and drummers combining culture, dance, and music for a spectacular display.

### Wadopana Celebration

**Wolf Point 406-768-7463** This celebration is the oldest traditional Pow Wow in Montana. The celebration is always the first weekend of August. The dancers participate for cash or prizes. It is a great weekend of fun and visiting among the elders and younger generations. They have special ceremonies for naming of individuals.

### Rocky Boy's Annual Pow Wow

**Box Elder 800-823-4478** The Pow Wow is an event of Indian Heritage and tradition. Dance, costume and drumming competitions are the featured events. Competitions last throughout the weekend with all ages participating. Exciting, excellent cultural demonstrations as well as ethnic and traditional food offered daily.



## *Mathematics Grade 9 – Pow Wow Trails (continued)*

### Heart Butte Celebration

**Heart Butte 406-338-7521** This annual four day celebration takes place the second week in August and is 26 miles south of Browning in the community of Heart Butte, which is located on the Blackfeet Reservation. Heart Butte is one of the oldest traditional communities of the Blackfeet Reservation.

### Hays Pow Wow

**Hays 406-353-2205** The Hays Pow Wow is one of the area's finest outdoor celebrations and features a whole weekend full of dancing and singing by natives from throughout the United States and Canada. Many dancing contests for all ages (from tiny tots to Golden Age!) Hand drum singing contests also!

### Crow Fair and Rodeo

**Crow Agency 406-638-3700** From every western state and Canada, Indians come to camp along the Little Bighorn River to form the Teepee Capital of the World. There are spectacular parades each morning featuring authentic costumes, an all-Indian rodeo with cowboys from all over the nation, pari-mutuel wagering, and a wild horse race.

### Ashland Labor Day Powwow

**Ashland 406-784-2883** The arbor on U.S. Highway 212 between Ashland and the St. Labre Mission comes alive during the Labor Day weekend. Drummers and dancers from many tribes participate. Booths are set up for food vendors and Native American art and beadwork. Giveaways, gourd dancing and hand games are part of the festivities.

### Poplar Indian Days

**Poplar 406-768-7623** Poplar Indian Days is a Pow Wow, which is a celebration of native culture and traditions through dancing, food, crafts, and fellowship with one another. All members of the General Public are welcome to participate or be a spectator. Poplar Indian Days will always be held during Labor Day weekend.

### Chief Plenty Coups Day of Honor

**Pryor 406-252-1289** This day of cultural sharing at this great Crow chief's home includes speakers, drummers and dancers; tours of the chief's house, medicine spring, and gravesite; and concludes with a buffalo feast. A craft fair is also being planned. All people are invited to participate in the Chief's great legacy.

### North American Indian Alliance Pow Wow

**Butte 406-782-0461** Butte hosts this annual celebration of Native American Dancing, food and music. There are vendors with Native American jewelry, beadwork and other traditional items.

### Native American Awareness Week

**Pablo 800-883-5344** Flathead Indian Reservation schools and the public share Salish, Kootenai, and Pend d'Oreille crafts, history, demonstrations, and exhibits by tribal departments during this week. Enjoy drumming, singing, and native dancing. Taste traditional fry bread and dry meat.

### Last Chance Community Pow Wow

**Helena 406-439-5631** The Last Chance Community Pow-Wow is held at the end of Native American Week in September. The Pow wow is a spiritual celebration to join together in dancing, singing, visiting old friends, and making new friends. Come on out and we will show you a wonderful time.

### American Indian Council of MSU Pow-Wow

**Bozeman 406-994-3751** Pow wows are an important contemporary expression of American Indian heritage. A Pow Wow is a Native American festival featuring traditional dancing, drumming and singing and traditional foods and arts and crafts. Various kinds of honor ceremonies and other spiritual ceremonies.

### Kyi-Yo

**Missoula 406-243-5403** The Kyi-Yo Celebration seeks to enhance cultural awareness on campus and create a more diverse atmosphere in our community. Kyi-Yo members strive to give people pride in their own culture as well as an appreciation of each other's unique identities.



Montana  
Office of Public Instruction  
Denise Juneau, State Superintendent

opi.mt.gov

# Mathematics

## Model Teaching Unit

### Reservation Land Areas

Created by: Deanna Reynolds

Grade 9/10 – Approximate Duration: 100 minutes

#### Stage 1 Desired Results

#### Established Goals:

**Number Sense and Operation Mathematics Content Standard 1:** A student, applying reasoning and problem solving, will use number sense and operations to represent numbers in multiple ways, understand relationships among numbers and number systems, make reasonable estimates, and compute fluently within a variety of relevant cultural contexts, including those of Montana American Indians.

- **1.2 Estimation and Reasonableness:** Select and apply appropriate estimation strategies to judge the reasonableness of solutions to problems including those computed on a calculator. Demonstrate correct use of order of operations.
- **1.5 Metric and Standard Measurement:** Use metric and standard units of measurement in relevant scientific and cultural situations, including those of Montana American Indians, compare and convert within systems, and use appropriate technology.

**IEFA: Essential Understanding 4:** Reservations are lands that have been reserved by the tribes for their own use through treaties, statutes, and executive orders and were not “given” to them. The principle that land should be acquired from the Indians only through their consent with treaties involved three assumptions:

- Both parties to treaties were sovereign powers.*
- Indian tribes had some form of transferable title to the land.*
- Acquisition of Indian lands was solely a government matter not to be left to individual colonists.*

#### Understandings:

*Students will understand...*

- how the size of one reservation has reduced over a period of years.
- how to make accurate measurements to calculate the area of a figure.
- how to calculate the area of irregular shapes by determining the common polygonal shapes that make up the irregular shape.
- how to convert square centimeter or square inch areas to the number of square miles by using the scale provided on the map.
- how to convert a square mile area into acres.

#### Essential Questions:

- What regular shapes could be used to estimate the area of the irregular shaped reservations?
- What measurements are needed in order to calculate the area of each reservation?
- Is there more than one approach that could be used to convert the map area into acreage area?
- Is there any relationship between the number of enrolled members and the land area set aside for their reservation?
- Are the calculated acreage amounts reasonable in comparison with reported information?



<p><i>Mathematics Grade 9 – Reservation Land Areas (continued)</i></p> <ul style="list-style-type: none"> <li>• how to compare calculated results to given results and check for reasonableness.</li> </ul>	
<p><i>Students will be able to...</i></p> <ul style="list-style-type: none"> <li>• calculate areas of irregular shapes by breaking them down into regular shapes.</li> <li>• get accurate measurements from maps to calculate the area.</li> <li>• use appropriate precision with measurements in reporting calculated area.</li> <li>• convert square units into the number of acres.</li> </ul>	<p><i>Students will know...</i></p> <ul style="list-style-type: none"> <li>• how to calculate the areas of irregular shapes.</li> <li>• how to discern if their calculated acreage is reasonable in comparison to that of reported values.</li> </ul>
<p style="text-align: center;"><b>Stage 2 Assessment Evidence</b></p>	
<p><b>Performance Tasks:</b> Worksheet with geometric shapes and Montana maps or Montana reservation maps. Worksheet with questions answered and turned in.</p> <p><b>Other Evidence:</b> Participation with group discussion. Observation of students taking measurements and calculating the areas of their reservation regions. Individual questioning of students.</p>	
<p style="text-align: center;"><b>Stage 3 Learning Plan</b></p>	
<p><b>Learning Activities:</b></p> <ol style="list-style-type: none"> <li>1. State the “Understandings” for the lesson.</li> <li>2. Introduction Activity: <ul style="list-style-type: none"> <li>• Show the “Shrinking Reservation” Map changing applet. After each year is selected ask the following questions: <ul style="list-style-type: none"> <li>• What is your estimation for the size change of the land area from the previous treaty?</li> <li>• What commonly used polygonal shape(s) do you believe would best estimate the reservation shape at each treaty change?</li> </ul> </li> </ul> </li> <li>3. Handout the Irregular Shapes worksheet. Have students calculate the area of each shape.</li> <li>4. Discuss answers of possible areas for each of the shapes. <ul style="list-style-type: none"> <li>• What are some ways the shape could have been sectioned into in order to estimate the area?</li> <li>• What measurements would be needed to calculate the area by the method you chose for estimating the area of your shape?</li> </ul> </li> <li>5. Have students complete questions #1 - #3 on the Reservation Land Acreage Worksheet.</li> <li>6. Handout MT Reservation maps. <ul style="list-style-type: none"> <li>• Divide class into seven groups. Each group needs a map of their reservation for each member.</li> <li>• Have groups calculate the area of their assigned reservation. Each group member calculates to check for accuracy.</li> </ul> </li> <li>7. Have students complete questions #4 - #6 on the Reservation Land Acreage Worksheet.</li> <li>8. Discussion with students regarding converting areas in square inches or square centimeters into the number of acres represented. <ul style="list-style-type: none"> <li>• What is the relationship of our selected unit of measure to that of acres?</li> </ul> </li> </ol>	

*Mathematics Grade 9 – Reservation Land Areas (continued)*

- How will we convert our area measurement into the number of acres represented? Have groups convert their areas into acreage amounts.
- 9. Have students complete question #7 on the Reservation Land Acreage Worksheet.
- 10. Handout the overview of each MT reservation and discuss if their acreage calculation was close to the reported acreage.
- What may be the contributing factors that resulted in different acreage amounts?
- How do we know if our calculations are reasonable?
- 11. Have students complete questions #8 - #9 on the Reservation Land Acreage Worksheet.
- 12. Discuss student responses to questions #8-#9.
- 13. Turn in worksheet.

**Summary:** You have calculated areas of irregular shapes and applied that knowledge to find an area of a reservation. You have also compared a calculated value to a documented value to determine reasonableness of the result. In addition to the mathematical focus, you have been introduced to some details of your specific tribe(s). You should now be able to apply your knowledge of irregular shapes to determine the area of any region.

**Materials/Resources Needed:**

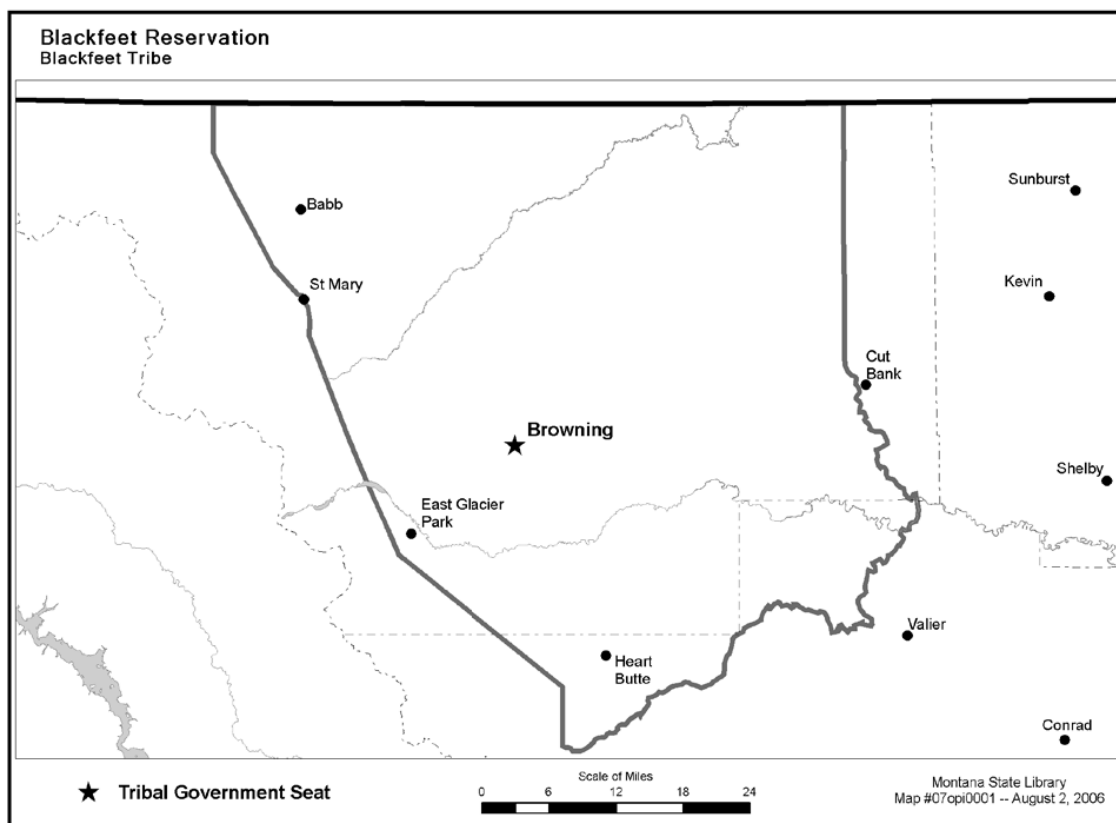
- Access to the website <http://www.trailtribes.org/greatfalls/shrinking-reservation.htm> for “The Shrinking Reservation” map series. At the trailtribes.org site, click on the map **Changes to Blackfeet Reservation** that is shown on the next page. It can be located at <http://www.trailtribes.org/greatfalls/sites/showonecontent.asp@contentid3612.htm>
- Contact information for the physical maps showing the similar changes in the Blackfeet Reservation are available at the website (<http://www.regionallearningproject.org>).
- Irregular Shape Worksheet for each student.
- Copies of MT reservation and copies of the overview of the seven Montana Reservations, one per group. (<http://www.opi.mt.gov/indianed/studentbackgroun.html> & <http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>).

**Extensions to the lesson: :** This lesson could be used in an Algebra 2 class while most juniors taking Algebra 2 will also be taking U.S. History. This lesson could be a collaborative lesson with the high school history teacher while the juniors are studying the reservation policies and different tribes. This way the students could be using information that they are discussing and researching along with the mathematics calculations to complete a unit. The students could be asked to find the enrollment number of tribal members of the reservation and any other item specific to their assigned reservation. The treaties that affected the reservation size should be noted in their presentations also. Students could create a power point presentation OR some visual type of poster and oral presentation to show all of their information to the entire class.

*Mathematics Grade 9 – Reservation Land Areas (continued)*

A second extension could be to focus only on the change in land area of the Blackfoot Territory through out the treaty years. The contact information to acquire those maps is listed in the resources above. Looking at the percentage change over the times of the treaty changes is quite interesting. Most of the written information about size of the other reservations is not as easily determined. Currently there are not maps available to illustrate those changes. The changes are explained in terms of landmarks but not located on any usable map.

## Blackfeet Reservation



<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

## Blackfeet Reservation

<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

### Location

The Blackfeet Reservation is located in Northeastern Montana along the eastern slopes of the Rocky Mountains. It is bounded on the north by the United States-Canadian boundary and extends 52 miles south to Birch Creek. The foothills of the Rockies form the western boundary and the eastern boundary approximates an imaginary line, which starts near the junction of Cut Bank Creek and the Marias River and extends northward. Within these boundaries, the land is mainly high, rolling prairies interspersed with rivers and creeks. The mountains found along the western border range in altitude between 4,400 to 9,600 feet. Browning, the gateway to Glacier National Park, is an incorporated town on the reservation. It has been the headquarters of the Blackfeet Indian Agency since 1894 and is the principal shopping center on the reservation. Other communities located throughout the reservation include East Glacier, Babb, St. Mary, Starr School, and Heart Butte.

### Population

Enrolled Members living on or near the Blackfeet Reservation 8,485

Enrolled Members living off the Blackfeet Reservation 6,633

Total number of enrolled Tribal members 15,118

### Land Status

Total acres within the Reservation's Boundary 1,525,712 acres

Individually allotted lands 701,815.57 acres

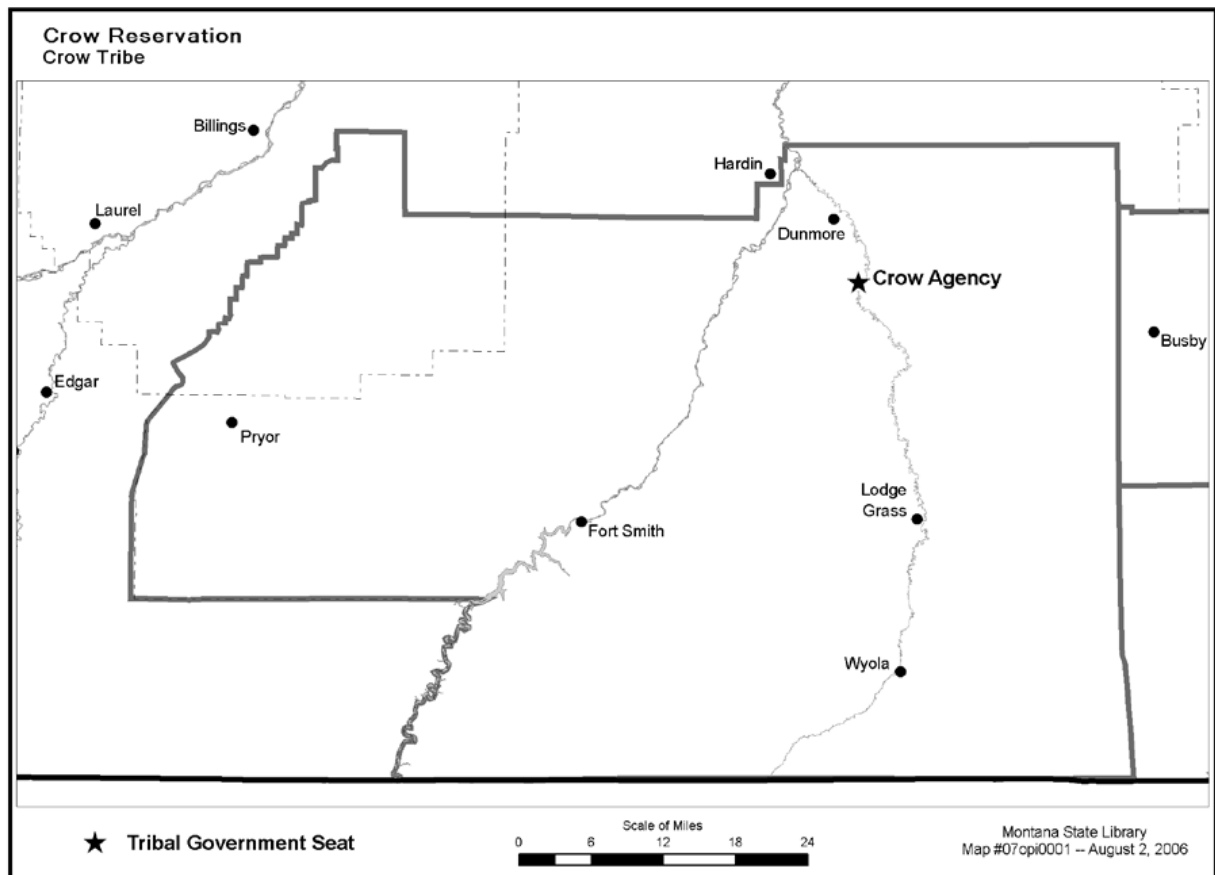
Tribally owned lands 311,174.98 acres

Government lands 1,654.46 acres

Fee title or state lands 511,067.10 acres

Presently, the land is used for ranching, farming, oil and gas development, and harvesting timber. The principal crops are wheat, barley, and hay. It is believed that traditional territorial lands of the Blackfoot Confederacy extended from the North Saskatchewan River south to Yellowstone Park, their western boundary being the Rocky Mountains and extending to the eastern boundary of Montana following the Missouri River.

## Crow Reservation



<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>



## Crow Reservation

<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

### Location

The Crow Indian Reservation encompasses a 4,989 square mile area or 3.8 million acres mainly in Big Horn County, in south central Montana. The Crow Reservation is nearly the same size as the entire state of Connecticut. The Crow Reservation is divided into six districts for cultural and Crow governmental purposes. They are: Reno, Lodge Grass, Pryor, St. Xavier, Wyola, and Black Lodge districts. The Crow Reservation has three mountainous areas, the Big Horn Mountains and the Pryor Mountains to the south/southwest, and the Wolf Teeth Mountains to the southeast. These mountain ranges meet the plains and valleys producing varied topography. In addition to the high mountains, the reservation includes gravely or stony slopes, broad hilltops with soils generally capable of supporting and maintaining excellent vegetative cover, level and productive irrigated valleys along the Big Horn and Little Big Horn Rivers and Pryor Creek, deep canyons, and extensive areas of rolling plateau. The nearest service center is Hardin, Montana, immediately adjacent to the reservation's northern boundary where restaurants, shops, and motels are available. However, the biggest retail and business center for the Crow Tribe is Billings located 90 miles north of Crow Agency, Montana.

### Population

Tribal Members living on or near the Crow Reservation 7,153

Tribal Members living off the Crow Reservation 3,180

Total number of enrolled Tribal members 10,333

### Land

Total acres within the Reservation's boundary 2,464,914 acres

Individually allotted lands 1,166,406 acres

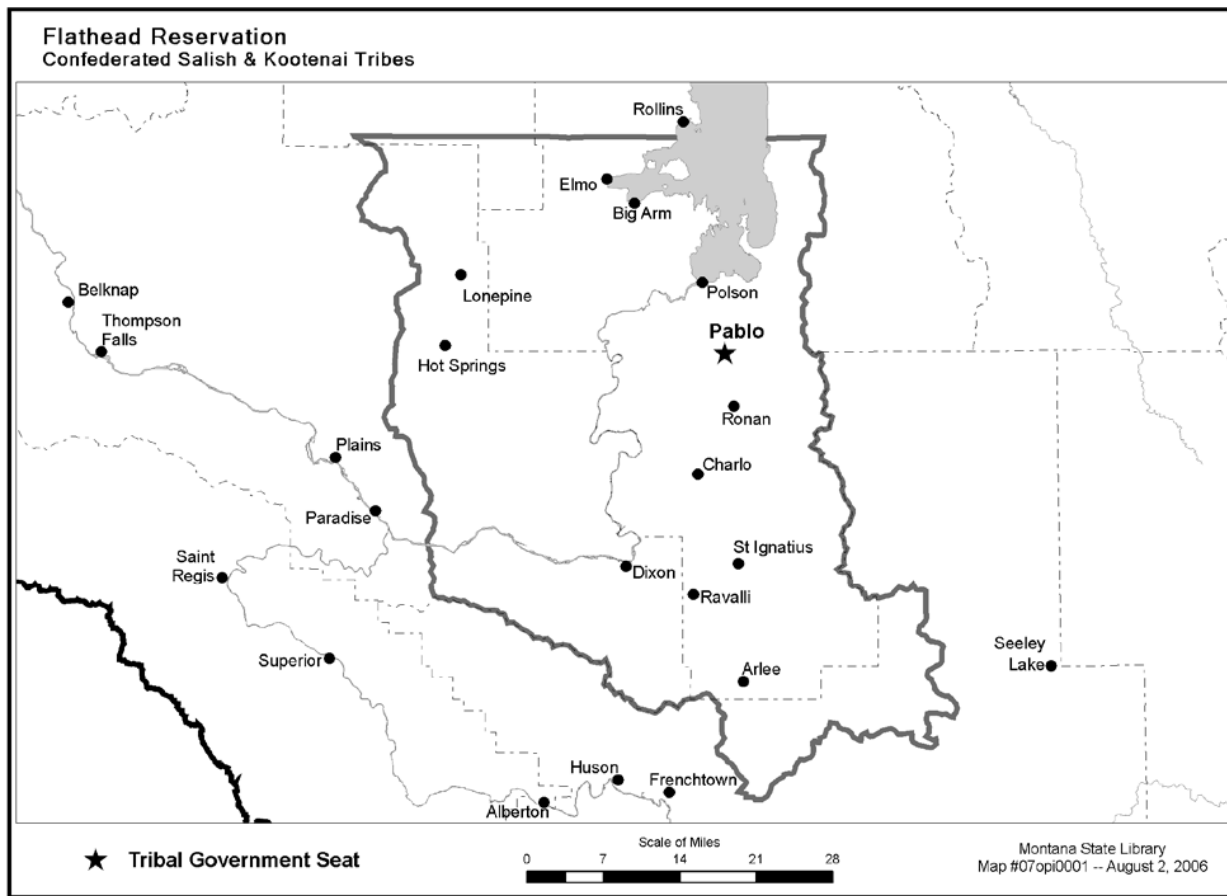
Tribally owned lands 404,172 acres

Fee title or state lands 709,167 acres

Government lands 1,135 acres

In 1973 over 31 percent of the land was owned by non-Indians. The Crow Indians operate only a small portion of their irrigated or dry farm acreage and only about 30 percent of their grazing land. More authority for the leasing of land without supervision has been extended to the Crow Tribe than to any other Indian tribe in Montana. Special legislation regarding Crow land in 1920, known as the Crow Act, and subsequent modifications occurring in 1926, 1948, and 1949, allowed most Crow tribal members to contract independent leases for individually owned land.

## Flathead Reservation



<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

## Flathead Reservation

<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

### Location

The Flathead Indian Reservation is located in northwestern Montana on the western slope of the Continental Divide. The exterior boundaries of the reservation include portions of four counties—Flathead, Lake, Missoula, and Sanders. The Flathead Reservation land base consists of approximately 1,243,000 acres. The eastern border of the reservation is at the top of the Mission Range of the Rocky Mountains. Flathead Lake and the Cabinet Mountain Range are to the north; the Lower Flathead River runs through the heart of the reservation; and to the west are the Salish Mountains and rolling prairie lands.

### Population

The three tribes of the Flathead Indian Reservation are the Salish, Pend d'Oreille, and Kootenai. Seliñ is the proper name for the Salish, who refer to themselves as Sqéliö—the People. The Salish have often been referred to as “Flatheads,” but this name is a misnomer and, in actuality, there are no Flatheads. Qæispé is the proper name for the Pend d'Oreille. The aboriginal name of the Kootenai Tribe is Kutanaxa, a name that means “licks the blood” in reference to a traditional hunting custom. The term Ktunaxa describes the Kootenai political sovereignty as a nation and all citizens who identify themselves as Kootenai. “Ksanka” refers to the name of the Ktunaxa band of the Flathead Reservation. Ksanka translates, “Standing Arrow,” which is a traditional warring technique. The tribes today are known by the contemporary title of The Confederated Salish and Kootenai Tribes. For the purposes of this document and for reader understanding, the terms Salish, Pend d'Oreille, and Kootenai will be used. After the reservation period, when lands were allotted and then subsequently opened to homesteading in 1910, many non-Indians moved to the reservation. The influx of homesteaders and the continuing movement of non-Indians onto the reservation have resulted in the Confederated Salish and Kootenai People being the minority population on their own reservation. Presently there are many Indian people from other tribes that live on the Flathead Reservation. Many are attending Salish Kootenai College or Kicking Horse Job Corps. Some have intermarried with tribal members and live among the community with their families. Both Salish Kootenai College and local K-12 public schools have identified over 40 different tribal nations represented within the student populations. There are 6,961 enrolled members of the Confederated Salish and Kootenai Tribes. Of this population, 4,244 live on the reservation.

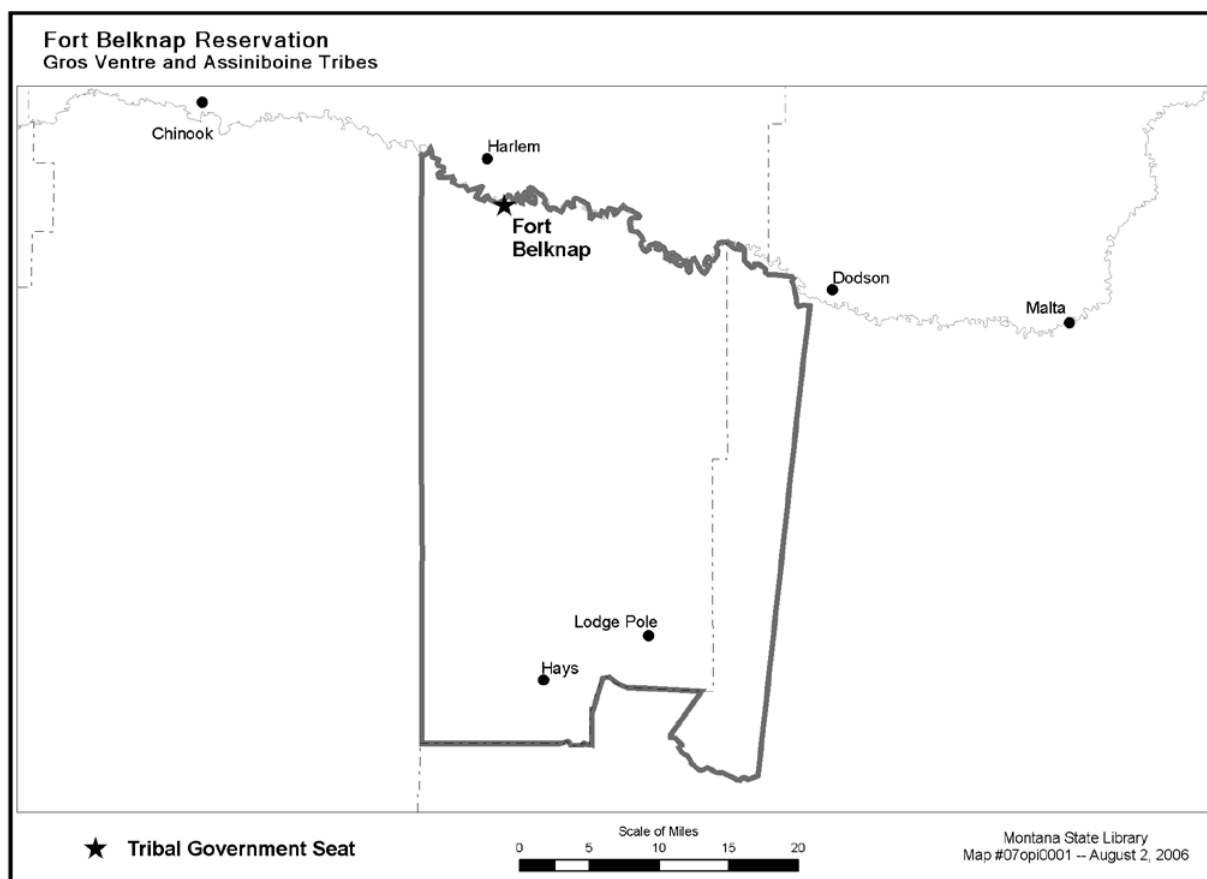
### Land

The Hellgate Treaty of 1855 created the Flathead Reservation. The treaty defined the boundaries of the reservation. A formal survey by the government actually diminished the reservation both on the northern and southern ends. The lands that remained as reserved by the treaty are approximately 1,243,000 acres. This land base was soon to change, however, with the passing of the Allotment Act (Dawes Act) of 1887, and the subsequent Homestead Act that opened the reservation to non-Indians in 1910. This resulted in the tribes becoming the minority landowners on their own reservation. While much of the prime agricultural land remains in non-Indian hands, the tribes have been aggressively buying back land. At this time they have become the majority landowners at roughly 56 percent. Following is a breakdown of the current status of reservation lands:

#### Area in Acres Status

613,273.50 Tribal Trust	76,159.25 Water
58,728.98 Tribal Fee	22,466.46 Federal
39,940.56 Individual Trust	40,742.57 State
4,248.89 Off-Reservation Tribal Fee	346.42 Town sites
466,480.67 Fee	

## Fort Belknap Reservation



<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

## Fort Belknap Reservation

<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

### Location

The Fort Belknap Reservation is located in north central Montana, south of the Milk River, within Phillips and Blaine Counties. Nearly 92 percent of the reservation is found in Blaine County with the remainder located along the western edge of Phillips County. The reservation's boundaries contain an area of approximately 675,336 acres. In addition, there are 29,731 acres of tribal land outside the reservation's boundaries. The north to south boundary extends 40 miles in length. The width is approximately 26 miles. Most of the northern portion of the reservation consists of the glacial plains and alluvial bottomlands. The southern portion of the reservation drains into the Missouri River and consists of rolling grasslands, river "breaks," and two principle mountain ranges, the Bearpaws and the Little Rocky Mountains. These mountains reach an elevation of approximately 6,000 feet.

### Population

Enrolled members living on or near the Fort Belknap Reservation 5,771

Enrolled members living off the Fort Belknap Reservation 1,532

Total number of enrolled tribal members 7,303

There are also Indians from other tribes, mostly Chippewa and Cree, living on the reservation, although they have no interests in tribal assets. Over the years, the reservation's resident Indian population has been decreasing. Some of the decline is due to the rural-urban shift, but a large proportion is a result of young people seeking off-reservation employment and educational pursuits.

### Land Status

Total acres within the reservation's boundaries 645,576 acres

Individually allotted lands 406,533 acres

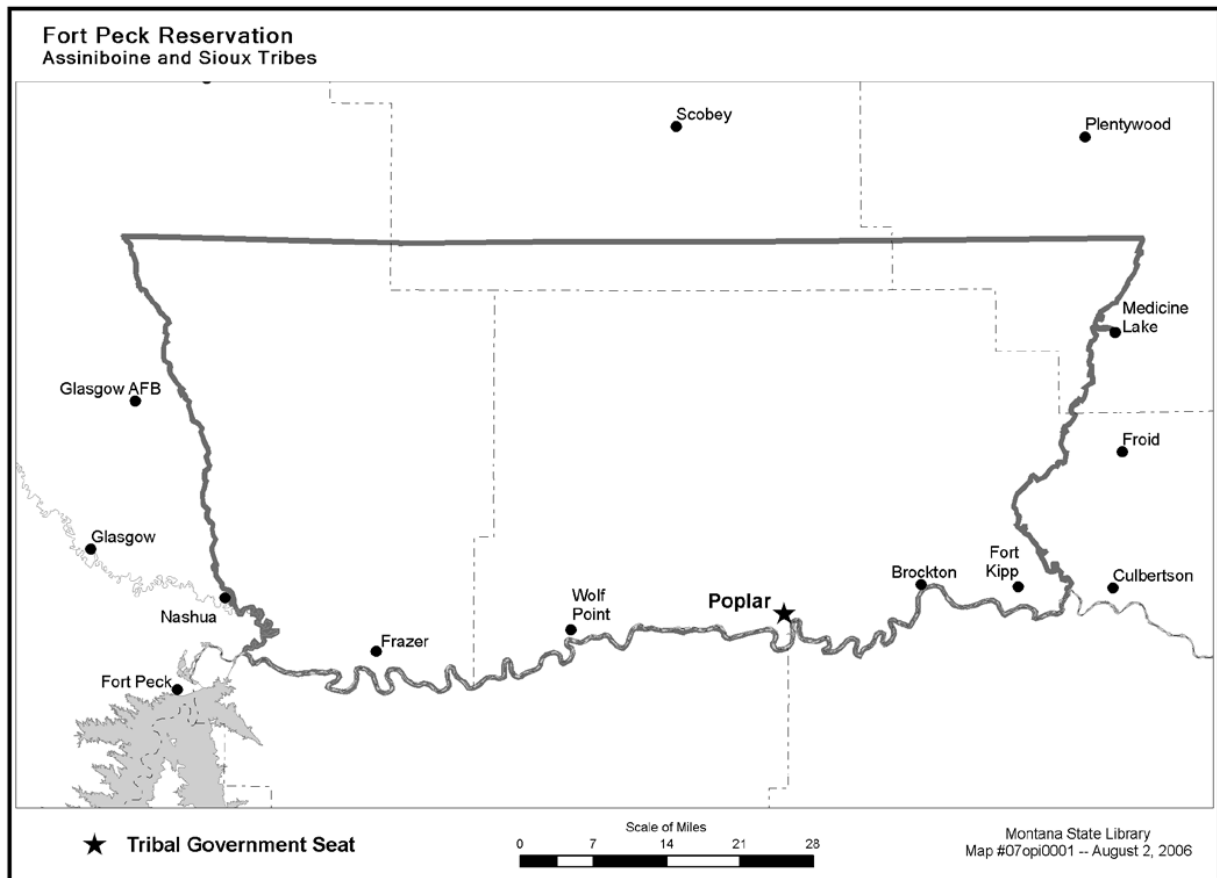
Tribally owned lands 210,954 acres

Fee title or state lands 19,000 acres

Government lands 592 acres

Non-Indian owned 9,000 acres

## Fort Peck Reservation



<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

## Fort Peck Reservation

<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

The Fort Peck Indian Reservation is home to a number of different Nakoda (Assiniboiné), Dakota, and Lakota (Sioux) communities that stretch along northeast Montana's Hi-Line from the Big Porcupine Creek to the Big Muddy Creek. The reservation, Montana's second largest in terms of land area, consists of 2,093,318 acres of which just under half is owned by individual tribal members or held in common by the Assiniboiné and Sioux Tribes. Linguistically, the Nakoda, Dakota, and Lakota are related. Sometime in the late 16th century they resided in the region between the Mississippi River and Lake Superior. As pressure from eastern tribes increased, the Nakoda split from the other Dakota and Lakota groups and moved north into Cree country. Today, bands of Nakoda, Dakota, and Lakota reside in Frazer, Oswego, Wolf Point, Poplar, Brockton, Riverside, and Ft. Kipp. These communities lay along the Missouri River's north bank, the reservation's southern boundary. Wolf Point, with a population of 4,000, is the largest town on the reservation and serves as the reservation's commercial center. Poplar, the next largest community, has a population of 3,200. Poplar is also the center of tribal government. The Ft. Peck Tribes, the BIA, and the Indian Health Service are headquartered there, as well as a number of other federally funded programs. The nearest primary trade centers are Billings, Great Falls (both approximately 300 miles from the reservation), and Williston, North Dakota, which lies some 75 miles east of Poplar.

### Population

Enrolled Sioux members 6,962

Enrolled Assiniboiné members 4,209

Total Ft. Peck tribal members 11,171

There are close to 1,000 members of other tribes living on Ft. Peck Reservation. One of the largest non-enrolled tribal groups is the Chippewa from the Turtle Mountain Reservation in North Dakota. The next largest non-enrolled group is the Assiniboiné from Ft. Belknap followed by individuals from the Three Affiliated Tribes (Mandan, Hidatsa, and Arikara) in North Dakota, and a number of Canadian Assiniboiné.

### Land Status

Total reservation acreage 2,093,124

Total tribal acreage 413,020

Total individually allotted acreage 516,092

Total fee simple or state acreage 1,164,012

The Ft. Peck Tribes have instituted an active policy of land acquisition. Over the past 20 years the Tribes have acquired over 19,000 acres. Ft. Peck, like most reservations, experienced the allotment policy, which resulted in the loss of just over half of tribal land holdings. Although the Dawes Act was enacted in 1887, it wasn't until the early 1900s when Ft. Peck was allotted. By 1922 the allotment process was near completion and lands not allotted were opened up for homesteading by non-Indians. Again, like most reservations, much of the better cropland passed into non-Indian hands. During the Depression, many non-Indian farms failed and the government repurchased the lands. An Act of Congress returned much of that land to the tribes in 1975.

## Little Shell Tribe of Chippewa Indians of Montana

<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

### Location

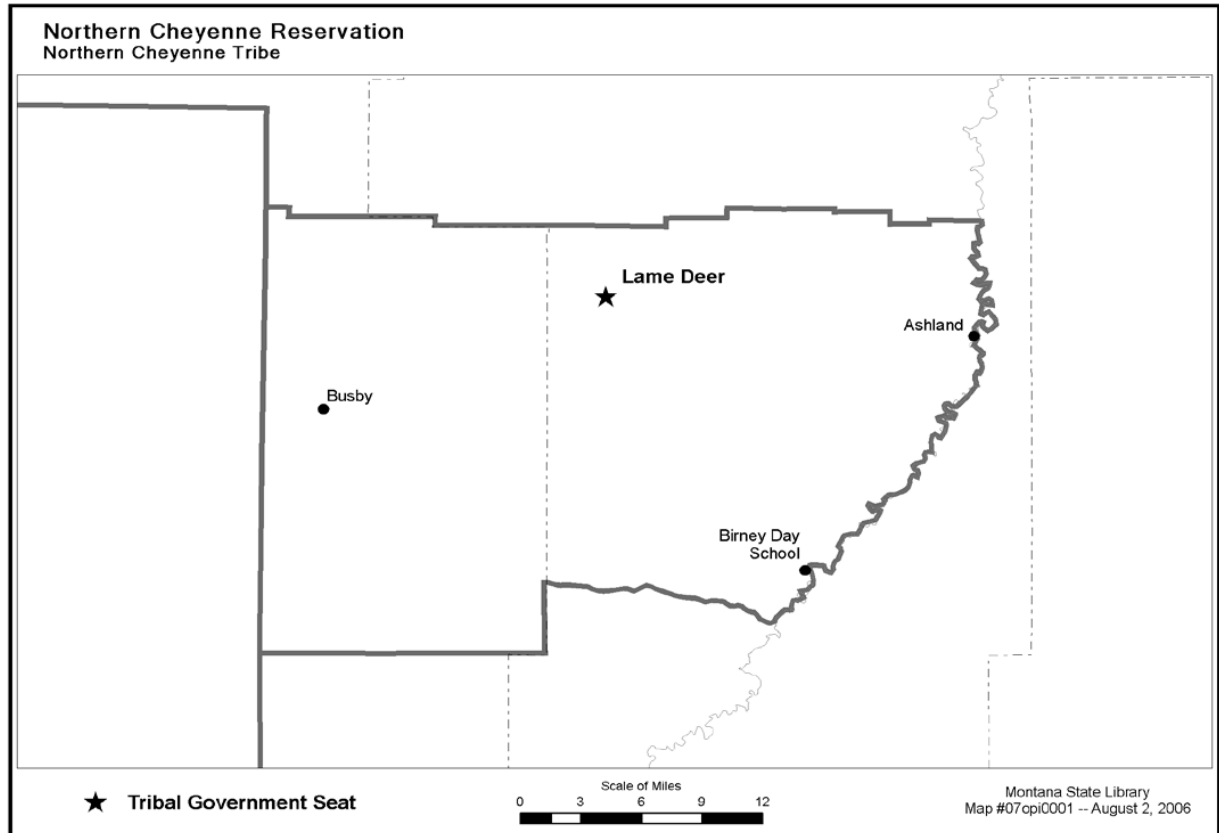
The Little Shell Chippewa Tribe is without a reservation or land base and members live in various parts of Montana. There are population concentrations in Great Falls, Havre, Lewistown, Helena, Butte, Chinook, Hays, Wolf Point, Hamilton, and Billings, as well as numerous other small communities in the state. Because the tribe has been without a land base for over 100 years, many members and their descendants live outside of Montana. Many changes are expected during the next decade as federal recognition is implemented.

### Population

The name of the tribe is: "The Little Shell Tribe of Chippewa Indians of Montana," and it is often shortened to "Little Shell." The name "Métis" (pronounced may-tee) is often used, meaning "middle people" or "mixed blood." The term Métis or more correctly Métifs, was first used during the 18th and 19th centuries, but at that time it identified a specific Northwest society with its own culture and economic traditions, living in the areas of the Red River, the Saskatchewan River, Turtle Mountain, North Dakota, and the area of present day Winnipeg and Pembina, North Dakota. A further discussion regarding this group is found in the subsequent section titled Ethnography and Historical Background. The current population of enrolled tribal members in Montana is approximately 3,850 and that number has not changed much in recent years. The tribe maintains only a rented office with volunteer staff, but continues to struggle for federal recognition. The Métis number in the thousands in the United States and south central Canada, and there are many unenrolled Little Shell people in Montana. Exact population numbers are not available. In the mid 1800s the tribe was numbered at several thousand in the Red River-Pembina region. At that time there was no formal enrollment procedure, no reservation and thus no documented population figure. After the 1892 renegotiation of the Treaty of 1863, (the infamous 10 cent treaty) many of the Métis, including the Band of Chippewa under Little Shell, were left without a land base or reservation, and many became nomadic.



## Northern Cheyenne Reservation



<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

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<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

### **Location**

The Northern Cheyenne Reservation, situated in southeastern Montana, lies within the counties of Big Horn and Rosebud. The Crow Reservation borders it on the west. The reservation consists of open ponderosa-pine plateau and valley country with an annual rainfall of approximately 16 inches. The topography ranges from about 4,800 feet to a low of a little less than 3,000 feet. The reservation headquarters and the center for business activities and population are in Lama Deer. The reservation itself is divided into five districts; Busby, Lama Deer, Ashland, Birney, and Muddy.

### **Population**

Total number of enrolled tribal members Approximately 7,374

Even though there are over 7,000 enrolled members, about 4,199 members live on the reservation scattered through the five district communities. There is also a relatively small population of non-Indians and other tribal members living on the reservation.

### **Land Status**

Total acres within the reservation's boundary 444,774.50 acres

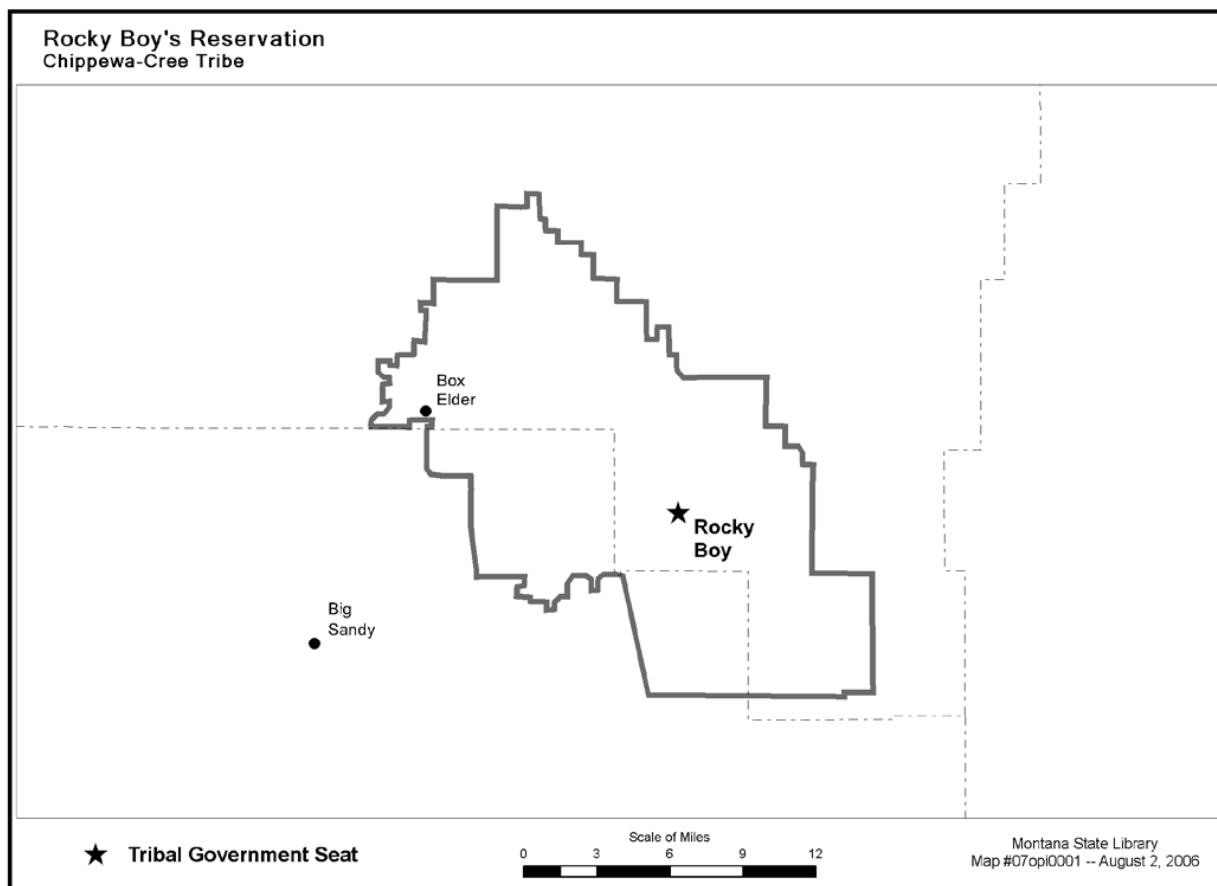
Individually allotted lands 113,277.70 acres

Tribally owned lands 326,546.81 acres

Fee title or state lands 4,827.70 acres

Non-Indians own about 30 percent of the fee or state lands on the Northern Cheyenne Reservation. The Tribal Council has selected a Land Acquisitions Committee whose primary policy is directed to the purchase of land into Tribal ownership. The Committee thus assures that Indian land is retained in Indian ownership.

## Rocky Boy's Reservation



<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

## Rocky Boy's Reservation

<http://www.opi.mt.gov/pdf/IndianEd/Resources/MTIndiansHistoryLocation.pdf>

### Location

The Rocky Boy's Indian Reservation is located in north central Montana, taking in portions of both Hill and Choteau counties. The reservation lies 90 miles south of the United States-Canadian border near the boundary separating the provinces of Alberta and Saskatchewan. The city of Havre (pop. 12,000) is located 26 miles to the north. U.S. Highway 87 between Havre and Great Falls intersects the reservation at Box Elder. Reservation roads total 216 miles with 62 providing well-paved, easy access to major points throughout the reservation. Airport facilities in Havre and Great Falls provide commercial airline services. Rail service, including Amtrak, is available in Havre on the main east/west line of the Burlington Northern Railroad; a south spur adjoins the reservation. Mt. Baldy, Mt. Centennial and Haystack Mountain are the more prominent landmarks found within the boundaries of the reservation. All three maintain significance in one way or another for the Chippewa Cree. East Fork and Bonneau Dams are also popular recreational areas. There is no town site on the reservation. The community of Rocky Boy is truly rural in every sense of the word. Rocky Boy's Agency is the hub of all reservation activity and serves as headquarters for the Chippewa Cree Tribe. The Rocky Boy Reservation is the smallest reservation in the state of Montana and the last to be established.

### Population

The reservation's unusual name comes from the leader of a band of Chippewa Indians. Translated from the Chippewa language it means Stone Child, but the original translation was lost and the name Rocky Boy evolved. The reservation was established by Executive Order in April of 1916, when Congress set aside 56,035 acres for the Chippewa and Cree Bands of Chief Rocky Boy. In 1947 the reservation was expanded by 45,523 acres, bringing it to nearly its current size. None of the land has been allotted, though some individual assignments have been made. The ethnic origin of the residents of the Rocky Boy's Reservation has remained complex, with the reservation becoming home to a diverse group of Cree, Chippewa, Métis, and Assiniboine peoples. The Cree represent one of the largest Native American groups in North America. While primarily residing in Canada today, a group of Cree settled in northern Montana after the Riel Rebellion in 1885. Led by Little Bear, these Cree eventually, after some three decades, became associated with a band of landless Chippewa under the leadership of Stone Child or Rocky Boy. The principal use of lands within the reservation is grazing and dry land farming. There is no substantial industry with the exception of a few small family-owned businesses. Although the reservation is isolated from larger metropolitan areas, community residents are avid participants in church; community and school related activities, such as basketball games. This extreme isolation also accounts for the rich cultural heritage continuing on the Rocky Boy's Reservation.

### Land

Rocky Boy's Reservation was established by Executive Order in 1916. Along with the passage of the 1934 Indian Reorganization Act, the Chippewa Cree Tribe had the opportunity to acquire the remaining land base, which consisted of area farm operations that had been abandoned during the depression era, thereby bringing the reservation land base to the current 122,259 surface acres. Rocky Boy's Reservation is unallotted and is held in trust for the entire tribe. The reservation's resource base consists of farm and range lands, minerals, timber stands, and the

*Mathematics Grade 9 - Reservation Land Areas (continued)*

Bear Paw Mountains which sustain wild game, fish, waterfowl, and springs and creeks that converge to form seven major drainages. The reservation has three distinct topographic zones including the Bear Paw Mountains in the southeastern portion, central rolling foothills and semi-arid plains in the north. The reservation is also split by Hill County covering the northeast and Choteau County covering the southwest portion of the reservation. Reservation topography is dissected, showing glacial plains and volcanic outcroppings. Small perennial streams arise in the Bear Paw Mountains, cutting steep sloped valleys. Elevation on the reservation ranges from 2,500 on the plains to 6,916 feet on top of Baldy Mountain. The average annual precipitation ranges from 10 inches at the lower elevations to 20 plus inches at the higher elevations. Temperature extremes are commonly from 110 degrees to -35 degrees Fahrenheit. Winters are long and cold and the roads are narrow and treacherous, particularly in the winter months.

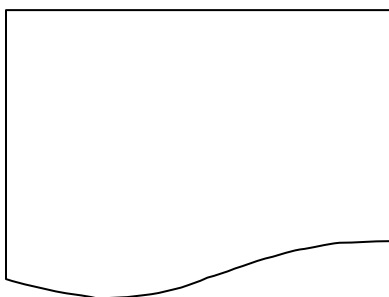
### Irregular Shapes

Name \_\_\_\_\_

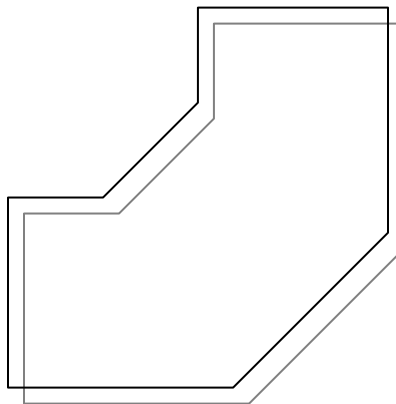
Below are several irregular shapes. Please complete the following for each:

- Decide how to sketch commonly used polygons over the top of the irregular shape to best estimate the area of the figure. **Draw** the division lines in the diagram.
- Identify the formula(s) of the common polygonal shapes you will use to calculate the estimated area.
- Find the appropriate measurements needed to calculate the area of the shape. Indicate your unit of measure in your calculations.

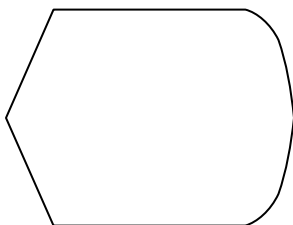
1.



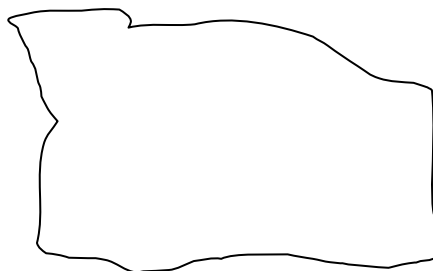
2.



3.



4.



### Irregular Shapes (Key)

Below are several irregular shapes. Please complete the following for each:

- Decide how to sketch commonly used polygons over the top of the irregular shape to best estimate the area of the figure. **Draw** the division lines in the diagram.
- Identify the formula(s) of the common polygonal shapes you will use to calculate the area.

**Most of the shapes provided can easily be divided into rectangular, triangular, and trapezoidal regions. There will need to be discussed that approximation is necessary when drawing in the divisions as very few divisions of irregular shapes work well.**

**Rectangle:**  $\text{base} \times \text{height}$

**Triangle:**  $0.5 \times \text{base} \times \text{perpendicular height}$

**Trapezoid:**  $0.5 \times (\text{short base} + \text{long base}) \times \text{perpendicular height}$

- Find the appropriate measurements needed to calculate the area of the shape. Indicate your unit of measure in your calculations.

#### Approximate Areas:

1.  $2.6 \text{ in}^2$
2.  $2.7 \text{ in}^2$
3.  $1.4 \text{ in}^2$
4.  $2.5 \text{ in}^2$

## Reservation Land Acreage Worksheet

Name \_\_\_\_\_

Reservation Assigned \_\_\_\_\_

1. What are some ways common polygonal shapes were used to estimate the area of the irregular shapes?
2. Did some the common polygonal shapes used to estimate the area require more measurements than the shapes you used?
3. How did you decide what unit of measure you would use? Examples: Measure to the nearest  $\frac{1}{4}$  inch,  $\frac{1}{2}$  inch or tenth of a centimeter. You should write your estimated area answer using the same corresponding units. How would your answer have differed if you had changed your nearest unit of measure?
4. What shapes did you use estimating the area of your reservation?
5. Was your area comparable to the rest of your group members? Was your value larger or smaller than the other members?
6. What is your group's agreed upon area of your reservation? Explain how you determined which value to use.



*Mathematics Grade 9 - Reservation Land Areas (continued)*

7. Calculate your reservation's acreage using the appropriate unit conversion.
  
  
  
  
  
  
  
  
  
  
8. How does your acreage calculation compare to the reported acreage amount? Where might the differences have come from?
  
  
  
  
  
  
  
  
  
  
9. Write a convincing argument as to whether your calculated acreage should be accepted. Your argument needs to address the percentage of difference between the calculations and the reported value.

## Reservation Land Acreage Worksheet

### Suggested Teacher Answers

1. What are some ways the irregular shapes were divided to calculate the area?

**These will depend upon the student's individual choices. Most of the irregular shapes will be divided into rectangles, triangle, and trapezoids.**

2. Did some of the ways for dividing the shapes require more measurements than the way you divided your shapes?

**Once again this will depend upon what the students' choices were. Trapezoidal division does require a few more measurements.**

3. How did you decide the precision you should write your area calculation answer in? Does it matter what precision you use?

**It is very important that students remember to use the scale on their map in their measurements. Students were allowed the choice of measuring in inches or centimeters. They need to keep in mind that they can not report an answer of area more precise than their original measurements were in.**

**Example: triangle with sides 3.4 cm by 3.6 cm should report the area as 6.1 square centimeters NOT 6.12 square centimeters. Students tend to write down however many decimal values their calculator gives them instead of thinking of significant digits.**

4. What shapes did you use in calculating the area of your reservation?

**This will be specific to the shape of their assigned reservation and personal choice.**

5. Was your area comparable to the rest of your group members? Was your value larger or smaller than the other members?

**Answer dependent upon the results of their group. Variations will be from different division of the reservation and possibly the differences in measurement choice. No matter which way the group measured or if some group members used cm and others used inches. The final area results when converted to square miles and acres should be close.**

6. What is your group's agreed upon area of your reservation? Explain how you determined which value to use.

**Some groups may say that they used the average of their areas (especially if they all used the same unit for measuring).**

7. Calculate your reservation's acreage using the appropriate unit conversion.

**Here you may need to remind students of the unit conversions.**

1 inch = 2.54 cm

12 inches = 1 foot

5280 feet = 1 mile

1 square mile = 640 acres

For example: **Rocky Boy Reservation** 208 square miles for 133,120 acres

*Mathematics Grade 9 - Reservation Land Areas (continued)*

8. How does your acreage calculation compare to the reported acreage amount? Where might the differences have come from?

**The reported acreage of the Rocky Boy Reservation is 122,259 surface acres. There is a difference of 10,861 acres or an estimate that is 8.9% larger than reported. The differences are due from the estimation of the shapes the area was broken up into when calculating the acres. Depending upon the precision students measured, the error can increase as the area is a squared calculation.**

9. Write a convincing argument as to whether your calculated acreage should be accepted. Your argument needs to address the percentage of difference between the calculations and the reported value.

**The calculations were completed for a Montana Road Map and not the copies of maps included with the lesson. Reasonable square mile & acreage numbers by reservation:**

- **Rocky Boy:** 208 square miles for 133,120 acres. There are approximately 2,500 enrolled members of the Chippewa-Cree tribe living on the reservation
- **Crow:** 3,406 square miles for 2,179,840 acres. There are approximately 5250 (75% of the 7,000 enrolled members) that live on or near the reservation.
- **Northern Cheyenne:** 648 square miles for 414,720 acres. Approximately 5,000 Northern Cheyenne, along with members of other tribes and non-Native Americans that live on the reservation.
- **Flathead:** 2,062 square miles for 1,319,680 acres. Over 4,000 Salish, Kootenai, and Pend d'Oreille of nearly 7,000 members live on the reservation.
- **Fort Belknap:** 974 square miles for 623,360 acres. A combined enrollment of the Assiniboiné and Gros Ventre is about 2,800.
- **Fort Peck:** 3221 square miles for 2,061,440 acres. About 6,800 Assiniboiné and Sioux live on the Fort Peck Reservation.
- **Blackfeet:** 2,145 square miles for 1,372,800 acres. About 7,000 Blackfeet tribal members live on the reservation.



Montana  
Office of Public Instruction  
Denise Juneau, State Superintendent

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# Mathematics

## Model Teaching Unit

### Beading Patterns Using Reflections

Created by: Linda Engebretson and Doug Mack

Grade 10 - Duration 110 minutes

#### Stage 1 Desired Results

##### Established Goals:

**Geometric Reasoning Mathematics Content Standard 3:** A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant cultural contexts, including those of Montana American Indians.

**IEFA Essential Understanding 1:** There is great diversity among the 12 tribal Nations of Montana in their languages, cultures, histories and governments. Each Nation has a distinct and unique cultural heritage that contributes to modern Montana.

**IEFA Essential Understanding 2:** There is great diversity among individual American Indians as identity is developed, defined and redefined by entities, organizations and people. A continuum of Indian identity, unique to each individual, ranges from assimilated to traditional. There is no generic American Indian.

**IEFA Essential Understanding 3:** The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs.

Additionally, each tribe has its own oral histories, which are as valid as written histories. These histories pre-date the “discovery” of North America.

##### Understandings:

- Properties of figures using transformations.
- An isometry created by a reflection.
- Reflection of coordinates through the x and y axis using the preimage creating the image.

##### Essential Questions:

- How are the coordinates of the preimage being reflected?
- How is a reflection created?
- Why is a reflection an isometry?
- How are the x and y coordinates changed when reflected through the x axis or the y axis?
- What is the importance of the line of symmetry to a reflection?

*Mathematics Grade 10 – Beading Patters Using Reflections (continued)*

*Students will be able to...*

- identify the x and y coordinates of each point for the shape.
- reflect the coordinates through the x axis or the y axis.
- identify the pattern when reflecting the coordinates through the x or y axis.
- identify an isometry.
- identify the coordinates of the preimage and the image.
- identify the line of symmetry.

*Students will know...*

- the quadrants of the Cartesian graphing system.
- how to form a conjecture about the x and y coordinates after reflecting the coordinates through the axis.
- how to identify the line of symmetry.
- how to determine the image coordinates after reflecting preimage through the x axis or the y axis.

**Stage 2 Assessment Evidence**

**Performance Tasks:**

- Use graph paper to plot points and create a design to be reflected.
- Complete reflections, reflecting the preimage creating the image.
- Identify the coordinates of the image using the prime notation.
- Identify an isometry.
- Identify the line of symmetry.
- Develop a conjecture regarding the line of symmetry and the line connecting a preimage point with an image point.

**Stage 3 Learning Plan**

**Learning Activities:**

Before beginning this reflection project, have students observe examples of beadwork. Some Native American beadwork provides beautiful examples of rotations, translations and reflections.

Refer to the excerpt found on page 71 of *Native America in the Twentieth Century: An Encyclopedia*, which was distributed to all Montana public school libraries, to find background information on the history and cultural importance of beadwork and beads.

For additional information regarding beadwork see the Smithsonian National Museum of the American Indian Web site titled, *Identity by Design - tradition, change, and celebration in native women's dresses*.

[http://www.nmai.si.edu/exhibitions/identity\\_by\\_design/IdentityByDesign.html](http://www.nmai.si.edu/exhibitions/identity_by_design/IdentityByDesign.html)

Before beginning the transformation, find an image from the website above that includes transformation and reflection. The pattern will be used for steps 11-17.

**Procedure:**

1. Each student needs 3 sheets of graph paper, ruler, and a minimum of four different colored pencils.
2. Draw the x and y axis in the center of the graph paper.

*Mathematics Grade 10 – Beading Patterns Using Reflections (continued)*

3. Plot the following points and draw the line segment AB. Next, reflect through the x axis by using the definition of reflection: transformation that uses a line that acts like a mirror, with an image reflected through the line.  
A(2, 4) B(4, 2)
4. Draw the image (the line A'B') formed by reflecting the points.
5. Discuss the following questions as a class: What is the line of symmetry? (Answer: the x axis) Does the line of symmetry always have to be the x or the y axis? (Answer: no, it could be any line, horizontal, vertical or  $y = x$ .) Is this an isometry? (Answer: yes, because it preserves length.) (isometry: A transformation that preserves length, angle measure, parallel lines, and distance between points)
6. Students should form a conjecture after plotting the points and reflecting the line through the x or y axis. Their conjecture should contain the following:
  - a. if (x,y) is reflected through the x-axis, its image is the point (x, -y)
  - b. if (x,y) is reflected through the y-axis, its image is the point (-x, y)
7. Have students write their conjecture on the graph paper they are using for this first reflection.
8. Discuss with the class: (Have students answer in class)
  - a. What happened to the x coordinate as the point was reflected about the x axis?
  - b. What happened to the y coordinate as the point was reflected about the x axis?
  - c. What happened to the x coordinate as the point was reflected about the y axis?
  - d. What happened to the y coordinate as the point was reflected about the y axis?
9. Have students draw segment AA', using the preimage point A and the image point A'.
10. Discuss with the class how the line of symmetry and the segment AA' are oriented with each other? (Answer: Students should recognize that they are perpendicular to each other and that the segment AA' is bisected by the line of symmetry.)
11. Using their second piece of graph paper, have students plot the following points forming a geometric figure, the preimage:  
A(0, 0)                      B(0, 3)                      C(-1, 2)                      D(-3, 0)  
E(-3, 4)                      F(-9, 4)                      G(-3,0)
12. Connect the following points forming a polygon: AB, BC, CD, ED, EF, FG, GA.
13. Using the conjecture they formed in the discussion from steps 7 and 8, reflect these points about the x axis forming the image. Remember to use the pattern (x, y)  $\rightarrow$  (x, -y) if reflecting about the x axis and (x, y)  $\rightarrow$  (-x, y) if reflecting about the y axis.
14. Label the image points using the prime notation: A'.

*Mathematics Grade 10 – Beading Patterns Using Reflections (continued)*

15. Connect the image points as the preimage points were connected above.
16. Next, reflect the preimage points about the y axis forming another image, labeling the new image points with double prime notation:  $A''$ .
17. Reflect the new image through the x axis, labeling the image point with the triple prime notation:  $A'''$ .
18. Color the pattern using different colors.
19. Using the third piece of graph paper, try the next reflection and color it in using different colored pencils. Here are the points to form the preimage for the next reflection:  
A(0, 2) B(-4, 4) C(-4, 1) D(-7, 1) E(-6, 0) F(-7, -1) G(-4, -1) H(-4, -4) I(0, -2)  
Squares inside the polygon: (-1,1) (-3,1) (-3,-1) (-1,-1)
20. Connect the following points: AB, BC, CD, DE, EF, FG, GH, HI and the points for the square inside the polygon.
21. Now reflect these points across the y axis forming the image. Remember the rule for reflection through the y axis: (x, y) then (-x, y).
22. Label the image points using the prime notation  $A'$
23. Connect the image points which will create an isometry (An isometry preserves length, angle measures, parallel lines and distance).
24. Color the design using different colors.
25. Now create your own design.

**At the end of this lesson students will:**

- Plot points and identify the x and y coordinates
- Reflect points through the x axis or the y axis and identify the pattern
- Apply the reflection to create the pattern
- Understand the project is an isometry

**Teachers resources:**

The web address below provides information about the history and cultural value of native women's dresses. The site also provides examples of beautiful beadwork and allows one to see the geometry involved in this art form.

[http://www.nmai.si.edu/exhibitions/identity\\_by\\_design/IdentityByDesign.html](http://www.nmai.si.edu/exhibitions/identity_by_design/IdentityByDesign.html)

*Mathematics Grade 10 – Beading Patters Using Reflections (continued)*

**Materials:**

- Graph paper (1 cm squares, 8 ½ x 11)
- Colored pencils
- Rulers

**Extension of this project:**

- Design or pattern and object to be beaded (watch band, key chain, pen cover, or barrette)
- Seed beads (a variety of colors)
- Thread
- Needles
- Leather (chamois works well)
- Cloth to wrap the object and beads into





Montana  
Office of Public Instruction  
Denise Juneau, State Superintendent

opi.mt.gov

# Mathematics Model Teaching Unit

## Tipi Geometry & Trigonometry

Created by: Marcia Welliever

Grade 10 - Approximate duration: 150 minutes

### Stage 1 Desired Results

#### Established Goals:

**Geometric Reasoning Mathematics Content Standard 3:** A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant cultural contexts, including those of Montana American Indians.

- **3.4 Indirect Measurement:** Determine measures of two- and three-dimensional objects and their elements using trigonometric ratios, proportionality, the Pythagorean Theorem, and angle relationships.

**IEFA Essential Understanding 1:** There is great diversity among the 12 tribal Nations of Montana in their languages, cultures, histories and governments. Each Nation has a distinct and unique cultural heritage that contributes to modern Montana.

**IEFA Essential Understanding 3:** The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs.

Additionally, each tribe has its own oral histories, which are as valid as written histories. These histories pre-date the “discovery” of North America.

#### Understandings:

*Students will understand...*

- how a tipi is constructed.
- how the Pythagorean Theorem relates to finding the height of a tipi.
- the relationships between radius, height, slant height, base area, lateral area and volume of a cone.

#### Essential Questions:

- How does the length of the tipi poles affect the radius, surface area and volume of a tipi?
- What are the reasons both past and present Indians use tipis?
- How do the properties of a conical solid relate to the advantages and disadvantages of constructing, living in, and transporting a tipi?

*Students will be able to...*

- find the volume, lateral area and surface area of a tipi.
- construct a net for the lateral area of a cone.
- relate the parts of a cone to a real-world object.

*Students will know...*

- the properties and measurable attributes of a cone.
- the Pythagorean Theorem.
- the formulas for volume, lateral area and surface area of a cone.

<p><i>Mathematics Grade 10 – Tipi Geometry and Trigonometry (continued)</i></p> <ul style="list-style-type: none"> <li>• use a trigonometric ratio to predict the angle of the tipi poles for a given pole length and radius or diameter.</li> </ul>	<ul style="list-style-type: none"> <li>• the definition of the trigonometric ratios.</li> <li>• traditional methods of putting up a tipi.</li> <li>• properties of circle.</li> </ul>
<p align="center"><b>Stage 2 Assessment Evidence</b></p>	
<p><b>Performance Tasks:</b> Research worksheet on tipis (both past and present). Math worksheets on computing measurements of a specific tipi. Create a model of a tipi using modern materials.</p> <p><b>Other Evidence:</b> Participation in discussion of research findings. Observation of student work. Individual questioning of students.</p>	
<p align="center"><b>Stage 3 Learning Plan</b></p>	
<p><b>Learning Activities:</b></p> <p>Day 1:</p> <ul style="list-style-type: none"> <li>• Begin the unit by taking 5 minutes to have your class brainstorm the various things they believe to be true about tipis. Write these on the board or on a reference sheet for later. Include any drawings.</li> <li>• Have the class complete the Tipi Research Worksheet using references from your school’s library and/or online resources. There are websites listed in the reference section, if you would like to direct their online work. (If using the older Linda Holley website, I would suggest focusing on the following sections: research area, historic photos &amp; drawings, tipi pattern drawings, linings &amp; ozans, and hide tipis.)</li> </ul> <p>Day 2:</p> <ul style="list-style-type: none"> <li>• Begin the day by reviewing the research questions and comparing to the brainstorm list from yesterday. Be sure to cross out any misconceptions or untruths from the list. Students will come back with a wide variety of answers, depending on which tribe or tribes they focused on. This is an important realization for them to have. Discussion of the conical properties as they relate to the tipi shapes (right vs. oblique for some tribes, 3-anchor vs. 4-anchor pole construction) is essential background knowledge for students to be able to complete the mathematical activities.</li> <li>• (Small groups) Have the students complete the Tipi Math Worksheet to compute the volume of the tipi and surface area of the cover. Be sure to have 3D cone models available to help students to visualize the relationship between the 3D real-life model and the 2D drawings on the worksheet.</li> </ul> <p>Day 3:</p> <ul style="list-style-type: none"> <li>• Project: Have the students use modern day materials and a scale of your choosing (ex: 1 inch = 1 foot) to construct a model of the tipi described on the Tipi Math Worksheet, or with permission they may want to construct a tipi using an alternate method they researched. Students can work in small groups or individually. Hand out copies of the scoring rubric. Choose a reasonable due date depending on how much you wish students to do in class and how much you expect them to do outside of class.</li> </ul>	

*Mathematics Grade 10 – Tipi Geometry and Trigonometry (continued)*

**Extension questions and activities:**

- How does changing the angle of the tipi poles with the ground affect the radius, height and volume of the tipi?
- Compute the volume of the tipi that would be created if the tipi poles were the same ( $l = 26$  ft), but the angle they create with the ground is changed to 40 degrees. How would this affect both the volume and the shape of the net?
- For a given surface area and tipi pole length, is there an angle with the ground that will maximize the volume of a tipi?
- Does the traditional 3 anchor method for putting up a tipi lead to the maximum possible volume?
- Find the maximum volume of a cone for a given slant height and radius or height.
- Determine if the Indians used an angle with the ground that maximizes the tipi volume for a given tipi pole length.
- Create pattern for tipi liner on model.

**Materials/Resources Needed:**

3-dimensional models of a cone

Crow tipi information from Little Big Horn College - <http://lib.lbhc.cc.mt.us/history/3.04.php>

Blackfeet tipi information from Glenbow Museum -

[http://www.glenbow.org/exhibitions/online/blackfoot/main\\_eng.htm](http://www.glenbow.org/exhibitions/online/blackfoot/main_eng.htm)

Please reference the following sites with prudence. Information found on these sites is contemporary and not completely verifiable as to its tribal authenticity.

<http://www.tipis.org/>

<http://www.tipis-teepees-teepees.com>

**Tipi Research Worksheet**  
**(\*Note: tipi, tepee, and teepee are interchangeable)**

Using textbooks or internet sources, research the following questions. Your responses are due tomorrow.

1. Which of the Indian Tribes in Montana traditionally used tipis and why?
2. Did any tribes outside of Montana use tipis? If so, what do they all have in common?
3. Most tipi shapes are shown as a right cone with a circular base. Some tribes used slightly adapted shapes. What are they, and what are the advantages or disadvantages to those shapes?
4. Most tipis were covered with tanned buffalo hides pieced and sewn together. Find a diagram or make a drawing to show the net or pattern used to create traditional tipis.
5. Traditional tipis were constructed using poles, typically made of lodge pole pine. Research the number of poles used and methods of erecting the structure. Did all Indian tribes use the same number of poles and the same method? Make a flow chart showing pictures, diagrams and/or verbal descriptions for one method of construction.
6. How did the Indians secure the tipi covers over the poles and to the ground?
7. What methods did the Indians use for temperature control during the various seasons of the year? Which of these methods are still used in modern tipis?
8. How did the Indians control smoke inside the tipi and keep weather outside the tipi?
9. Most modern Indians live in houses, but some also have tipis. For what purposes are modern tipis used? What adaptations with modern materials have been developed and what are their advantages?

Extension questions:

10. Modern homes use drywall, paint, wood or wallpaper as interior decoration. Did traditional tipis have interior decorations? If so, what kind?
11. How were tipis transported? Did the method of transportation have any effect on the size of the tipi used?
12. What furnishings and layouts were used in tipis?
13. Did tipis have a kitchen for cooking?

## Tipi Math Worksheet

- One method of constructing tipis is to lash the tops of 3 anchor poles together and place the bottoms of those 3 poles in a triangle where the distance between the poles is the same as the radius of the semicircular cover (some tribes like the Crow use 4 poles). Let the tipi poles overlap each other at the top by 6 feet. More poles (10-20) are laid around a circular base and intersect the anchor poles at the top to create the shell for the tipi. The semicircular cover is then lifted to the top of the tipi, billowed around the poles, and the flaps are overlapped and pinned together to create the tipi. (visit this site for pictures - [http://www.tipis.org/images/c\\_tipi\\_lg.jpg](http://www.tipis.org/images/c_tipi_lg.jpg))

Individually, or in small groups, complete the following:

- Make a drawing to show the 3-dimensional view of the tipi.
  - Label the intersection of the tipi poles as point V.
  - Label the base of the 3 anchor poles as points P, Q and R.
- If the tipi poles are about 32 feet long and they overlap on the top by 6 feet, what is the slant height of your cone? \_\_\_\_\_

What is the length of the radius of the semicircular cover for the tipi? \_\_\_\_\_

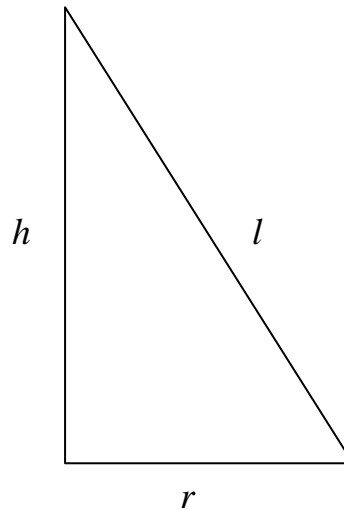
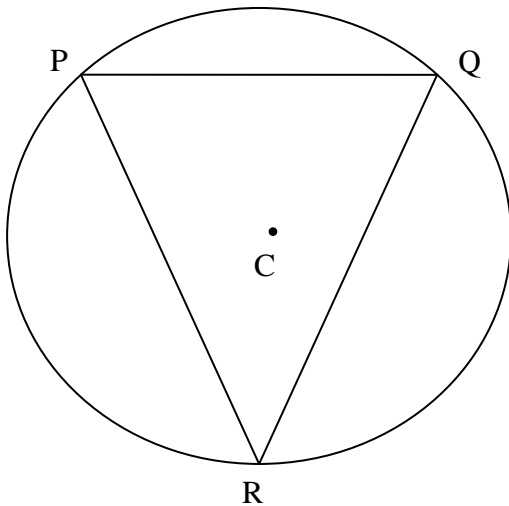
What is the distance between the bottoms of the anchor poles? \_\_\_\_\_

What kind of a triangle do the anchor poles make on the ground? \_\_\_\_\_

- Use the circle below to represent the base of the tipi. (Drawing not to scale) Let the vertices of the triangle represent the bottoms of the anchor poles and C is the center of the circular base.
  - Label the midpoint of side PQ as point M.
  - Draw in triangle CMP and label all of its sides and angles.
  - Justify how you found those sides and angles.
  - What is the radius of the base of the cone? \_\_\_\_\_
  - Label this radius length on your 3D drawing.
- Use the right triangle below to represent the one formed by the radius (r), height (h), and slant height (l) of the cone.
  - Use points from the previous drawings to label the vertices of the right triangle below.
  - What mathematical concept can you use to find h in the right triangle below?  
\_\_\_\_\_
  - Find and label the height of the cone (h) on your 3D drawing.
- Write the formula for the volume of a right circular cone. \_\_\_\_\_
- Use your formula to find the approximate volume of the tipi using appropriate units.  
\_\_\_\_\_
- Use one of the trigonometric ratios (sine, cosine, or tangent) to determine the angle that the anchor poles make with the ground. Label that angle on the right triangle below.

*Mathematics Grade 10 – Tipi Geometry and Trigonometry (continued)*

8. The lateral area of the cone tells us how much material is needed to create the cover of the tipi.
- Write the formula for the lateral area of a cone. \_\_\_\_\_
  - Use your formula to find the surface area of the tipi using appropriate units. \_\_\_\_\_



**Tipi Research Worksheet Answer Key**  
**(\*Note: tipi, tepee, and teepee are interchangeable)**

Using textbooks or internet sources, research the following questions. Your responses are due tomorrow.

14. Which of the Indian Tribes in Montana traditionally used tipis and why? **All of Montana's Indian tribes, including the Sioux, Assiniboiné, Crow, Blackfeet, Northern Cheyenne, Salish-Kootenai, Chippewa-Cree tribes used tipis. They were used because they were easy to transport, could be made from local materials, and were adaptable to Montana's varied climate conditions.**
15. Did any tribes outside of Montana use tipis? If so, what do they all have in common? **Many tribes outside of Montana used tipis. The tribes that used tipis were the Plains Indians who lived and traveled on the Great Plains of North America.**
16. Most tipi shapes are shown as a right cone with a circular base. Some tribes used slightly adapted shapes. What are they, and what are the advantages or disadvantages to those shapes? **Some Crow and Blackfeet constructed tipis that were in the shape of an oblique cone with a more oval-shaped base. The off-center shape made it possible to arrange the interior of the dwelling differently, make the tipi larger and was a very sturdy construction technique.**
17. Most tipis were covered with tanned buffalo hides pieced and sewn together. Find a diagram or make a drawing to show the net or pattern used to create traditional tipis. **(See tipi math worksheet.)**
18. Traditional tipis were constructed using poles, typically made of lodge pole pine. Research the number of poles used and methods of erecting the structure. Did all Indian tribes use the same number of poles and the same method? Make a flow chart showing pictures, diagrams and/or verbal descriptions for one method of construction. **Many tribes used a 3-pole construction technique, where 3 poles were anchored together at the top to begin setup and then other poles were laid over the top to build the support system for the tipi cover. Many Crow and Blackfeet tribes, however, typically used 4-poles to begin construction, and had a more oblique cone tilted away from the door opening.**
19. How did the Indians secure the tipi covers over the poles and to the ground? **The tipi covers, traditionally made of tanned buffalo hides, were lifted onto the anchor poles and wrapped around the pole "skeleton". Many tipi covers were secured along the slant height with small pegs that went through the covers like button holes. They could be secured to the ground through straps and pegs at the bottom or by placing rocks around the perimeter of the base to secure the tipi against winds and drafts.**

20. What methods did the Indians use for temperature control during the various seasons of the year? Which of these methods are still used in modern tipis? **Campfires could be built just off-center towards the door opening when heat was needed. Some tipis were constructed with ozans or liners which were like vertical walls added around the perimeter of the tipis' interiors to control drafts. They were typically hung from a rope at about head or shoulder height. During the warm months, the liners could be eliminated and part of the tipi cover could be lifted to create air movement to cool the interior of the tipi. All of these methods are still used in modern tipis, with the addition of modern portable heating devices.**
21. How did the Indians control smoke inside the tipi and keep weather outside the tipi? **A flap near the top could be narrowed or widened to let smoke escape and control the amount of heat in the dwelling. A rain cap was sometimes added to the construction in wet weather. It allowed the smoke to escape, but “funneled” the rain away from the interior of the tipi. Door covers could be added to provide privacy and access through the oval-shaped opening, while preventing heat exchange.**
22. Most modern Indians live in houses, but some also have tipis. For what purposes are modern tipis used? What adaptations with modern materials have been developed and what are their advantages? **Modern Indians sometimes use tipis as informal summer dwellings in rural settings. Tipis are used as temporary dwellings during traditional or spiritual ceremonies, such as pow wow gatherings, like the Crow Fair. Many modern tipis are used by both Indian and non-Indian people. Modern updates of materials are canvas tipi covers that can be mildew-resistant, waterproof and weatherproof.**

Extension questions:

23. Modern homes use drywall, paint, wood or wallpaper as interior decoration. Did traditional tipis have interior decorations? If so, what kind? **Many tipis had painted ozans or liners that displayed Native American artistry.**
24. How were tipis transported? Did the method of transportation have any effect on the size of the tipi used? **Tipis could be transported by layering the tipi cover and interior belongings on top of the tipi poles and “dragging” them behind dogs or horses. When horses were used, it was possible to use much larger tipis.**
25. What furnishings and layouts were used in tipis? **Many Indian tribes used beadwork backrests as interior furnishings. These provided support and were a way to display artistic talent as well. Some tipi arrangements had buffalo hide beds which were scattered around the tipi perimeter wall on side away from the door opening.**
26. Did tipis have a kitchen for cooking? **In warmer months, most tribes had a cooking fire outside of the tipis. In cooler months, an interior fire could be used for both heating and cooking.**

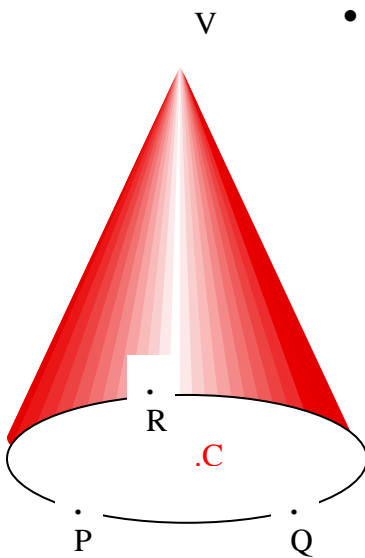


### Tipi Math Worksheet Answer Key

- One method of constructing tipis is to lash the tops of 3 anchor poles together and place the bottoms of those 3 poles in a triangle where the distance between the poles is the same as the radius of the semicircular cover. Let the tipi poles overlap each other at the top by 6 feet. More poles (10-20) are laid around a circular base and intersect the anchor poles at the top to create the shell for the tipi. The semicircular cover is then lifted to the top of the tipi, billowed around the poles, and the flaps are overlapped and pinned together to create the tipi.

Individually, or in small groups, complete the following:

- Make a drawing to show the 3-dimensional view of the tipi.
  - Label the intersection of the tipi poles as point V.
  - Label the base of the 3 anchor poles as points P, Q and R.



- If the tipi poles are about 32 feet long and they overlap on the top by 6 feet, what is the slant height of your cone? 26 feet

What is the length of the radius of the semicircular cover for the tipi? 26 feet

What is the distance between the bottoms of the anchor poles? 26 feet

What kind of a triangle do the anchor poles make on the ground? equilateral

Mathematics Grade 10 – Tipi Geometry and Trigonometry (continued)

6. Use the circle below to represent the base of the tipi. (Drawing not to scale) Let the vertices of the triangle represent the bottoms of the anchor poles and C is the center of the circular base.

- Label the midpoint of side PQ as point M.
- Draw in triangle CMP and label all of its sides and angles.

$$\begin{aligned} m\angle PMC &= 90 & PM &= 13 \text{ ft} \\ m\angle MCP &= 60 & MC &= \frac{13}{\sqrt{3}} = \frac{13\sqrt{3}}{3} \approx 7.5 \text{ ft} \\ m\angle CPM &= 30 & CP &= \frac{26}{\sqrt{3}} = \frac{26\sqrt{3}}{3} \approx 15 \text{ ft} \end{aligned}$$

- Justify how you found those sides and angles.
- What is the radius of the base of the cone?  $CP = \frac{26}{\sqrt{3}} = \frac{26\sqrt{3}}{3} \approx 15 \text{ ft}.$
- Label this radius length on your 3D drawing.

4. Use the right triangle below to represent the one formed by the radius (r), height (h), and slant height (l) of the cone.

- Use points from the previous drawings to label the vertices of the right triangle below.
- What mathematical concept can you use to find h in the right triangle below?

Pythagorean Theorem

- Find and label the height of the cone (h) on your 3D drawing.

9. Write the formula for the volume of a right circular cone.  $V = \frac{1}{3}\pi r^2 h$

10. Use your formula to find the approximate volume of the tipi using appropriate units.

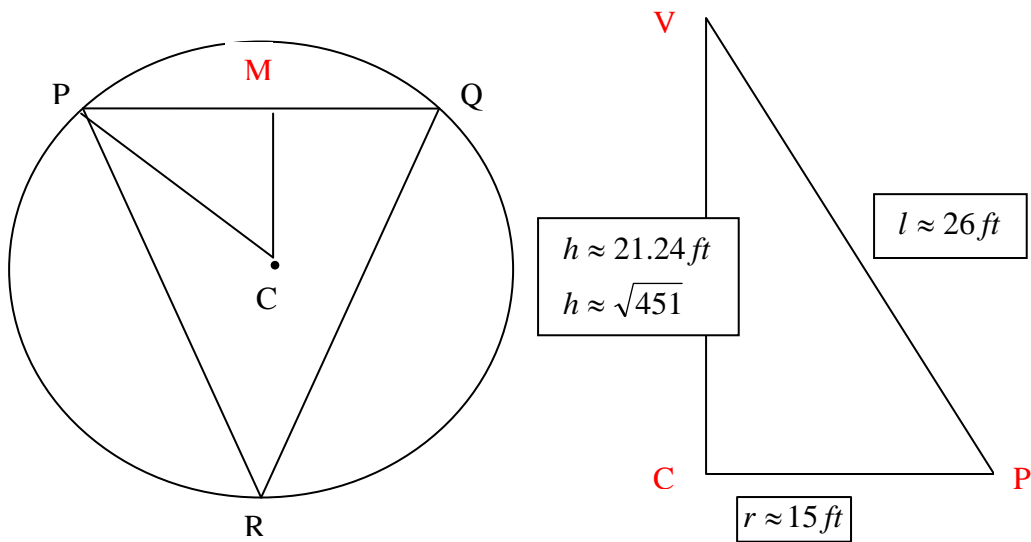
$$V = \frac{1}{3}\pi(15)^2(21.24) \approx 1086 \text{ ft}^3$$

11. Use one of the trigonometric ratios (sine, cosine, or tangent) to determine the angle that the anchor poles make with the ground. Label that angle on the right triangle below.

$$m\angle VPC = \cos^{-1}\left(\frac{15}{26}\right) \approx 54.8^\circ$$

12. The lateral area of the cone tells us how much material is needed to create the cover of the tipi.

- Write the formula for the lateral area of a cone.  $LA = \pi rl$
- Use your formula to find the surface area (no base) of the tipi using appropriate units.  $LA = \pi(15)(26) = 390\pi \approx 1225 \text{ ft}^2$



### Tipi Model Construction Project

Materials needed: Pattern paper. Canvas for tipi covers. Wooden dowels (small diameter) for tipi poles.

Round wooden toothpicks. Liner or ozan material (optional).

- Using a scale of \_\_\_\_\_, construct a paper template of the net (pattern) needed to construct a model of the tipi described in the tipi math worksheet. Label the pattern with any variables used on the math worksheet. Make a copy of your pattern to turn in to your teacher.
- Using one of your patterns, determine the size of canvas rectangle (to the nearest  $\frac{1}{8}$  yard) that you will need to construct your tipi cover. Confirm the size needed with your teacher.
- Using the appropriate amount of canvas, cut out your net.
- Determine how many tipi poles you wish to use and how many dowels you will need to construct adequate lengths.
- Use the small wooden toothpicks to lash together the tipi cover.
- Construct a door cover and show smoke flaps. (Liner or ozan are needed for a maximum score)
- Compute the lateral area and volume of the model constructed. Include all formulas used and show all mathematical work. Use appropriate labels. Include this sheet with your pattern page.

	Template for Holistic Scoring of Tipi project
5	Demonstrates complete understanding. All mathematical formulas and work are accurate. 3D model is neat and well-constructed, including door cover, smoke flaps, and liner.
4	Demonstrates considerable understanding. Minor errors in mathematical work. 3D model may have minor flaws or may be missing liner or door cover.
3	Demonstrates partial understanding. Wrong formulas used, but mathematical work is accurate. Pattern (net) used might not yield an accurate model. May have pattern, but no 3D model. Most requirements of task are included.
2	Demonstrates little understanding. Major mathematical errors. Model only partially constructed. Either mathematics is not completed or the model is not attempted.
1	Demonstrates no understanding. Major flaws in any work completed. Major portions of task are not completed.
0	No attempt



Montana  
Office of Public Instruction  
Denise Juneau, State Superintendent

opi.mt.gov

# Mathematics Model Teaching Unit

## Montana Native American Population

Created by: Deanna Reynolds

Grade 11 – Approximate Duration: 100 minutes

### Stage 1 Desired Results

#### Established Goals:

**Data Analysis Mathematics Content Standard 2:** A student, applying reasoning and problem solving, will use data representation and analysis, simulations, probability, statistics, and statistical methods to evaluate information and make informed decisions within a variety of relevant cultural contexts, including those of Montana American Indians.

- **2.1 Representing and Analyzing Data:** Select, create, and compare graphical or numerical representations of data sets using technology when appropriate. Reason about distributions using measures of central tendency and spread (e.g., percentiles, quartiles, inter-quartile range, and standard deviation).
- **2.5 Curve Fitting:** Model two-variable data using curve fitting with and without technology. Write an equation for a given model and decide when or if predictions based on this equation are valid.

**IEFA: Essential Understanding 4:** Reservations are lands that have been reserved by the tribes for their own use through treaties, statutes, and executive orders and were not “given” to them. The principle that land should be acquired from the Indians only through their consent with treaties involved three assumptions:

*I. Both parties to treaties were sovereign powers.*

*II. Indian tribes had some form of transferable title to the land.*

*III. Acquisition of Indian lands was solely a government matter not to be left to individual colonists.*

**IEFA: Essential Understanding 7:** Under the American legal system, Indian tribes have sovereign powers, separate and independent from the federal and state governments. However, the extent and breadth of tribal sovereignty is not the same for each tribe.

#### Understandings:

*Students will understand...*

- how to create a scatter plot to show the population trend of the American Indian Population on or around a reservation.
- how to create a scatter plot to show the total population trend on or around a reservation.

#### Essential Questions:

- What changes have occurred in the American Indian population over the years 2000-2006?
- What changes have occurred in the total population over the years 2000-2006?
- How do the changes compare for the two different populations?



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<p><i>Mathematics Grade 11 – Montana Native American Population (continued)</i></p> <ul style="list-style-type: none"> <li>• how to look for trends associated with the scatter plot and a model that fits the trend.</li> <li>• how to make predictions to the future population using their model.</li> <li>• how to calculate a residual value for a data set.</li> </ul>	<ul style="list-style-type: none"> <li>• Is there more than one type of model that could fit a data set?</li> <li>• How do we decide if a model is a reasonable model for a data set?</li> </ul>
<p><i>Students will be able to...</i></p> <ul style="list-style-type: none"> <li>• create a scatter plot representing the population data of the American Indians and total population for all Montana reservations and their related areas.</li> <li>• find a model that best fits the data either by hand or using appropriate technology.</li> <li>• make predictions using their model.</li> <li>• create a residual plot for a data set.</li> </ul>	<p><i>Students will know...</i></p> <ul style="list-style-type: none"> <li>• how to create a scatter plot and analyze the trend represented by the data.</li> <li>• how to use their model to make predictions.</li> <li>• how to use residual plots to decide if a model is a good fit.</li> </ul>

### Stage 2 Assessment Evidence

**Performance Tasks:** Graphs and models for data sets of American Indian and total population for MT reservations and their related areas. Worksheet with questions answered and turned in.

**Other Evidence:** Participation with group discussion. Observation of students' graphs and method of finding their model to fit the data. Individual questioning of students.

### Stage 3 Learning Plan

#### Learning Activities:

1. State the "Understandings" for the lesson.
2. Introduction Activity. Divide class into eight groups. Hand out Demographic & Economic information pamphlets for each Montana Reservation, as well as the pamphlet for the Little Shell tribe. Each student in the group receives one for the reservation or tribe they have been assigned. After students have had a few minutes to review their information, ask the class the following questions:
  - Are the values reported for all counties on the reservation or around the tribes? If not, why do you think this is the case? (This is a discussion generating question and will not have an absolute response.)
  - Does the American Indian population for the reported counties tend to increase or decrease?
  - Does the total population for the same reported counties tend to increase or decrease?
3. Complete questions #1- #4 on student worksheet.
4. Have each group post their scatter plots around the room so that they are visible for the entire class.
  - What type of model(s) best represents the data change?
  - With what we know about populations, what type of model best fits this trend?
5. Complete questions #5 - #6 on student worksheet.
6. Have each group write their model of best fit along side the corresponding scatter plot.
  - What is common about the given models?
  - What is different?

*Mathematics Grade 11 – Montana Native American Population (continued)*

7. Complete questions #7 - #10 on student worksheet.
8. Turn in worksheet.

**Summary:** You have created a graph to represent the change in American Indian population near reservations and total populations of the same area. After analyzing the trend represented by your graph, you determined a model that could be used to make predictions about the population in the next few years.

**Materials/Resources Needed:**

- Copies of the Demographic & Economic information for the seven MT reservations and the Little Shell tribe, for each student. The State of Montana site, (<http://www.ourfactsyourfuture.org/cgi/databrowsing/?PAGEID=4&SUBID=249>) offers very nice printable pamphlets to use.
- Graph paper or large paper to create scatter plot on. Scatter plots could be created on a type of technology that can be shown to all students via a view screen or Smartboard.
- Graphing utility or device capable of calculating the best fit model.

**Extensions:** This lesson could focus on the population percentage of American Indians by age group and the population percentage of Montanans by age group. The data can be modeled by a logistic function for both each tribe compared to the Montanan percentage. Discussing with students why the same type of model would not work for this data, could lead to interesting comments.

**Student Worksheet**  
**Montana Reservation Population**

Name\_\_\_\_\_

Reservation\_\_\_\_\_

1. Create a scatter plot representing the data for the American Indian Population for the years 2000-2006.
2. Explain the trend you see exists with your data sets?
3. Create a scatter plot representing the data for the total population for the years 2000-2006.
4. Does the same trend exist for the total population as did in the American Indian population? Compare and contrast the trends.
5. What possible model(s) would best fit your population? Determine the model for each of your data sets. Be sure to write your model in the correct form.
6. Determine the predicted population values for the years 2000-2006 using the models found in question #5. Use the predicted values to calculate the residuals for your data sets.
7. What does a negative residual value mean? What does a positive residual value mean?
8. Create a residual plot for the American Indian population and a separate residual plot for the total population. Do the residual plots indicate that your models are a reasonable model?
9. Use your model(s) to predict the population(s) of the American Indian and total for **2007**. Should we use our model to predict for the year **2012**?
10. Create a residual plot for a different type of model that “may” fit the data sets. Which model is the best fit after looking at your different residual plots?



**Student Worksheet (Key)**  
**Montana Reservation Population**

Name\_\_\_\_\_

Reservation\_\_\_\_\_

1. Create a scatter plot representing the data for the American Indian Population for the years 2000-2006.

**Scatter plots vary depending upon the tribe of focus. It is important that the plots are easily read by the entire class. The scatter plot could be created on a graphing calculator and then displayed on an overhead view screen or Smartboard if available.**

2. What trend do you see exists with your data set?

**Generally speaking the tribal populations are increasing. Some of the total populations are increasing and others are decreasing.**

3. Create a scatter plot representing the data for the total population for the years 2000-2006.

4. Does the same trend exist for the total population as did in the American Indian population? Compare and contrast the trends.

**This response will be dependent upon their specific tribe of focus.**

5. What possible model(s) would best fit your populations? Determine the model for each of your data sets. Be sure to write your model in the correct form.

**Some students will believe that the data looks very linear since it is for such a small amount of time. Others will believe that it represents an exponential model since we have an initial population and then percentage of change.**

6. Determine the predicted population values for the years 2000-2006 using the models found in question #5. Use the predicted values to calculate the residuals for your data sets.

7. What does a negative residual value mean? What does a positive residual value mean?

**A negative residual means the predicted value is larger than the actual value. A positive residual value means the predicted value is smaller than the actual value.**

*Mathematics Grade 11 – Montana Native American Population (continued)*

8. Create a residual plot for the American Indian population and a separate residual plot for the total population. Do the residual plots indicate that your models are reasonable?

**If the residual plot (x-value, residual) has the points randomly scattered above and below the x-axis, this graph indicates that a reasonable model has been selected.**

9. Use your model(s) to predict the population(s) of the American Indian and total for **2007**. Should we use our model to predict for the year **2012**?

**These values will depend upon the model used. Predicting outside of our data set is not a good practice, especially with an exponential model.**

10. Create a residual plot for a different type of model that “may” fit the data sets. Which model is the best fit after looking at your different residual plots?

**As stated in question #5, some students will try a linear model here.**

This table represents the possible exponential models for the American Indian Population by reservation and for the total population.

Reservation (American Indian Population)	Exponential Model
Fort Peck	$y = 6657.5 * 1.008^x$
Fort Belknap	$y = 3444.5 * 1.0009^x$
Blackfeet	$y = 8928.8 * 1.007^x$
Northern Cheyenne	$y = 10523.7 * 1.009^x$
Rocky Boy's	$y = 3738.1 * 1.01^x$
Flathead	$y = 9972.8 * 1.013^x$
Crow	$y = 11574.3 * 1.02^x$
Reservation (Total Population)	Exponential Model
Fort Peck	$y = 24278 * 0.99^x$
Fort Belknap	$y = 11490 * 0.987^x$
Blackfeet	$y = 19533.7 * 1.0004^x$
Northern Cheyenne	$y = 22042 * 1.002^x$
Rocky Boy's	$y = 22426.5 * 0.99^x$
Flathead	$y = 208630 * 1.012^x$
Crow	$y = 141820 * 1.011^x$

Here is some teacher Information regarding the total population and American Indian population by related areas. The percentage change of both the American Indian population and total population is indicated along with the average percentage of change by Reservation.

<http://www.ourfactsyourfuture.org/cgi/databrowsing/?PAGEID=4&SUBID=249>

Reservation	Year	American Indian Pop.	Percentage Change	Total Population	Percentage Change
<b>Fort Peck Related Areas:</b> <b>Counties of Daniels, Roosevelt, Sheridan, and Valley</b>	2000	6679	1.32%	24417	-1.45%
	2001	6767	-0.93%	24064	-2.09%
	2002	6704	0.81%	23562	-1.25%
	2003	6758	1.97%	23267	0.25%
	2004	6891	1.86%	23326	-1.02%
	2005	7019	-0.44%	23087	-1.62%
	2006	6988	(average) 0.76%	22712	(average) -1.20%
<b>Fort Belknap Related Areas:</b> <b>Counties of Blaine and Phillips</b>	2000	3502	-2.28%	11610	-2.93%
	2001	3422	0.15%	11270	-0.91%
	2002	3427	-0.03%	11167	-1.32%
	2003	3426	0.18%	11020	-1.23%
	2004	3432	1.19%	10884	-1.06%
	2005	3473	0.63%	10769	-0.52%
	2006	3495	(average) -0.03%	10713	(average) -1.33%

<b>Northern Cheyenne and Related Areas: Counties of Big Horn and Rosebud</b>	2000	10539	0.98%	22054	0.16%
	2001	10642	0.38%	22090	0.06%
	2002	10682	0.64%	22104	0.37%
	2003	10750	1.69%	22186	0.44%
	2004	10932	1.49%	22284	0.32%
	2005	11095	-0.23%	22355	-0.26%
	2006	11069	<b>(average)</b> 0.82%	22296	<b>(average)</b> 0.18%
<b>Rocky Boy's and Related Areas: Counties of Choteau and Hill</b>	2000	3736	1.98%	22643	-1.52%
	2001	3810	-0.45%	22299	-1.37%
	2002	3793	1.11%	21994	-0.70%
	2003	3835	1.36%	21840	0.17%
	2004	3887	1.21%	21877	-0.63%
	2005	3934	1.40%	21740	0.37%
	2006	3989	<b>(average)</b> 1.10%	21820	<b>(average)</b> -0.61%
<b>Flathead and Related Areas: Counties of Flathead, Lake, Missoula, Sanders</b>	2000	9970	1.46%	211048	-0.40%
	2001	10116	0.90%	210200	1.17%
	2002	10207	1.43%	212662	1.38%
	2003	10353	1.31%	215600	1.52%
	2004	10489	0.95%	218879	1.61%
	2005	10589	1.65%	222396	1.83%
	2006	10764	<b>(average)</b> 1.29%	226475	<b>(average)</b> 1.19%

<b>Blackfeet and Related Areas: Counties of Glacier and Pondera</b>					
	2000	8990	-0.50%	19671	-0.95%
	2001	8945	0.84%	19485	-0.18%
	2002	9020	0.62%	19450	0.10%
	2003	9076	1.18%	19469	0.75%
	2004	9183	0.73%	19615	-0.05%
	2005	9250	0.49%	19606	0.02%
	2006	9295	<b>(average)</b> 0.56%	19610	<b>(average)</b> -0.05%
<b>Crow and Related Areas: Counties of Big Horn and Yellowstone</b>					
	2000	11630	1.87%	142023	0.97%
	2001	11847	1.93%	143399	0.92%
	2002	12076	0.95%	144715	0.92%
	2003	12191	2.39%	146044	1.20%
	2004	12482	2.85%	147796	1.26%
	2005	12838	3.05%	149662	1.06%
	2006	13229	<b>(average)</b> 2.17%	151248	<b>(average)</b> 1.05%

**Student Information: Percentages used to create the Percentage Pyramids**

**Population Percentage by Age Group and Reservation:** The Little Shell Chippewa Tribe does have percentages by age group also, but is not included in this information.

Age Group	Montana Percent	Crow Reservation	Flathead Reservation	Fort Belknap Reservation	Fort Peck Reservation	Northern Cheyenne Reservation	Rocky Boy Reservation
0-4	6.1%	10.0%	7.1%	10.1%	8.6%	11.5%	12.0%
5-9	6.9%	10.2%	7.6%	11.1%	10.2%	12.1%	12.5%
10-14	7.7%	10.7%	8.8%	12.9%	11.1%	12.8%	12.5%
15-19	7.9%	10.3%	8.6%	11.3%	9.1%	10.6%	10.6%
20-24	6.5%	6.4%	5.2%	6.1%	5.5%	6.1%	7.7%
25-29	5.7%	5.3%	4.9%	4.9%	5.7%	6.6%	5.7%
30-34	5.8%	6.1%	5.4%	5.6%	5.4%	6.7%	6.2%
35-39	7.4%	7.6%	6.9%	7.6%	7.1%	6.6%	7.3%
40-44	8.4%	7.3%	8.0%	7.4%	8.3%	6.1%	6.4%
45-49	8.1%	6.3%	7.7%	5.8%	7.1%	5.2%	5.2%
50-54	6.8%	5.8%	6.6%	4.4%	5.4%	5.6%	3.6%
55-59	5.2%	4.2%	5.4%	3.3%	3.9%	3.4%	3.3%
60-64	4.2%	3.6%	4.6%	3.0%	3.2%	2.7%	2.2%
65-69	3.6%	2.4%	4.0%	2.4%	3.0%	1.8%	2.4%
70-74	3.3%	1.6%	3.7%	1.9%	2.5%	1.1%	1.1%
75-79	2.7%	1.3%	2.9%	1.3%	1.9%	0.9%	0.8%
80-84	2.0%	0.7%	2.0%	0.6%	1.6%	0.3%	0.5%
85+	1.7%	0.5%	1.9%	0.4%	1.3%	0.4%	0.2%

### Teacher Information

This information is the difference between the total percentage of Montanans and each tribe. A negative value indicates that the tribal percentage in that age group was less than that of all Montanans. The standard deviation value represents the standard deviation for that age group.

Age Group	Crow Difference	Flathead Difference	Fort Belknap Difference	Fort Peck Difference	Northern Cheyenne Difference	Rocky Boy Difference	Standard Deviation
0-4	3.9%	1.0%	4.0%	2.5%	5.4%	5.9%	0.01659851
5-9	3.4%	0.8%	4.2%	3.3%	5.2%	5.7%	0.01603296
10-14	3.1%	1.1%	5.3%	3.4%	5.1%	4.8%	0.01448941
15-19	2.4%	0.7%	3.4%	1.2%	2.7%	2.7%	0.00939335
20-24	-0.1%	-1.2%	-0.3%	-1.0%	-0.4%	1.2%	0.00783976
25-29	-0.3%	-0.8%	-0.8%	0.1%	0.9%	0.0%	0.00572292
30-34	0.3%	-0.4%	-0.2%	-0.4%	0.9%	0.4%	0.00483038
35-39	0.2%	-0.5%	0.2%	-0.3%	-0.8%	-0.1%	0.00375261
40-44	-1.1%	-0.3%	-1.0%	-0.1%	-2.3%	-1.9%	0.00793263
45-49	-1.8%	-0.4%	-2.3%	-1.0%	-2.9%	-3.0%	0.00935149
50-54	-1.0%	-0.2%	-2.4%	-1.4%	-1.3%	-3.2%	0.00964459
55-59	-1.0%	0.2%	-1.9%	-1.3%	-1.8%	-1.9%	0.00744209
60-64	-0.6%	0.4%	-1.2%	-1.0%	-1.5%	-2.0%	0.00748328
65-69	-1.2%	0.4%	-1.2%	-0.6%	-1.8%	-1.2%	0.00697562
70-74	-1.7%	0.4%	-1.4%	-0.8%	-2.2%	-2.2%	0.00917058
75-79	-1.4%	0.1%	-1.5%	-0.9%	-1.8%	-1.9%	0.00691851
80-84	-1.4%	-0.1%	-1.5%	-0.4%	-1.7%	-1.5%	0.00615795
85+	-1.2%	0.2%	-1.3%	-0.4%	-1.3%	-1.5%	0.00601474



Montana  
**Office of Public Instruction**  
Denise Juneau, State Superintendent  
In-state toll free 1-888-231-9393

# Mathematics Model Teaching Unit

## Seven Stars

Created by: Jessie Collins

**Grade 12—Approximate Duration: 50 minutes**

### Stage 1 Desired Results

#### Established Goals:

**Algebraic and Functional Reasoning Mathematics Content Standard 4:** A student, applying reasoning and problem solving, will use algebraic concepts and procedures to understand processes involving number, operation, and variables and will use procedures and function concepts to model the quantitative and functional relationships that describe change within a variety of relevant cultural contexts, including those of Montana American Indians.

- **4.3 Solving Systems of Equations and Inequalities:** Solve a variety of equations, inequalities and systems of equations and inequalities, justify the solution process, and interpret the solution in context.
- **4.5 Analyzing and Conjecturing with Models:** Given data or a problem situation, select and use an appropriate function model to analyze results or make a prediction with and without technology using cultural contexts, including those of Montana American Indians.

**IEFA Essential Understanding 3:** The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs.

Additionally, each tribe has its own oral histories, which are as valid as written histories. These histories pre-date the “discovery” of North America.

**IEFA Essential Understanding 6:** History is a story most often related through the subjective experience of the teller. With the inclusion of more and varied voices, histories are being rediscovered and revised. History told from an Indian perspective frequently conflicts with the stories mainstream historians tell.

#### Understandings:

*Students will understand...*

- how the Crow star stories and constellations correspond to the Euro-American constellations.
- how to use and manipulate logarithmic formulas to learn about individual stars’ properties.

#### Essential Questions:

- How is the brightness of stars determined?
- What is the brightest star we can see from earth?
- How do the stars in the Seven Stars constellation compare to the sun?



<p><i>Mathematics Grade 12 – Seven Stars (continued)</i></p> <p><i>Students will be able to...</i></p> <ul style="list-style-type: none"> <li>• use known data to calculate unknown values.</li> <li>• use the properties of logarithms to simplify and solve problems.</li> <li>• compare very large and very small numbers.</li> <li>• manipulate formulas/derive own formulas.</li> </ul>	<p><i>Students will know...</i></p> <ul style="list-style-type: none"> <li>• the story of the Seven Stars as told by a Crow Elder from Lodge Grass.</li> <li>• the definitions of absolute magnitude, apparent magnitude, luminosity and parsec.</li> <li>• the names of the stars in the Big Dipper/Seven Stars constellation.</li> </ul>
<p align="center"><b>Stage 2 Assessment Evidence</b></p>	
<p><b>Performance Tasks:</b> Worksheet with table containing star information, vocabulary, formulas, and star map. Worksheet with questions answered and turned in.</p> <p><b>Other Evidence:</b> Participation in classroom discussion. Observation of data calculation. Test over logarithms at the end of the unit/chapter. Individual questioning of students.</p>	
<p align="center"><b>Stage 3 Learning Plan</b></p>	
<p><b>Learning Activity</b></p> <ol style="list-style-type: none"> <li><b>1. State the “Understandings” for the lesson.</b></li> <li><b>2. Read aloud the Seven Stars excerpt from <u>The Stars We Know</u> by Timothy P. McCleary, pgs 69-71.</b> <ul style="list-style-type: none"> <li>• Discuss the names of the stars/planets/constellations, both the Crow as well as their commonly known names.</li> <li>• Site given in Resources</li> </ul> </li> <li><b>3. Handout worksheet with star table, formulas, vocabulary, and star map.</b> <ul style="list-style-type: none"> <li>• Read the stars’ names (check online for proper pronunciation if necessary) and note that the first eight stars are from the Seven Stars story.</li> <li>• Go over the vocabulary with the class, making sure to stress the difference between apparent and absolute magnitudes.</li> <li>• Go over the formulas and variables making sure students understand when/why to use each one.</li> </ul> </li> <li><b>4. Have students fill in the table, leaving Alioth blank (they will fill this in later).</b> <ul style="list-style-type: none"> <li>• Go over values together as a class; make sure everyone has the correct values in the table. Use the last formula for filling in the “Brightness” column.</li> <li>• Look at the values for the sun, Polaris, and Sirius and note how distance has an effect of the magnitude and brightness—just because a star has a higher absolute magnitude, it’s not necessarily brighter to viewers on Earth.</li> <li>• Discuss which stars seem to be the brightest—are any actually brighter than the sun? Polaris? Sirius? (Sample response: All of the stars in the table are many times brighter than the sun but much farther away so they don’t appear as bright to us on Earth. Polaris is the brightest star, being 2271 times brighter than the sun but is the farthest away of the stars in the table by over 300 light years. Although Sirius is visually the brightest star, all but Alcor and the sun are brighter when distance is factored into the comparison.)</li> <li>• Are there any stars that appear brighter to us on Earth but are really not as bright as the others? (Sample response: Sirius is the third dimmest when comparing brightness but is the most visible in the night sky. Look for stars that have absolute magnitudes that are more than 1.5 units higher than their</li> </ul> </li> </ol>	

*Mathematics Grade 12 – Seven Stars (continued)*

apparent magnitudes. The sun is the most obvious, going from being the most visible from Earth to having the highest absolute magnitude.)

**5. Hand out worksheet with questions.**

- Have students complete question 1 and 2—fill in the information for Alioth in the table. Go over answers.
  - Do these new values change the answers to Learning Activity 4? (Sample: Not really but it does move from being the third most visible to having the 6<sup>th</sup> lowest absolute magnitude.)
- Have students complete #3 on worksheet (you may want to do this one together on the board or give them a hint to start out)
  - Discuss answer—from earth, which star appears brighter? (Sirius appears brighter but Polaris is really much brighter when distances are equal)
- If time, have students complete #4 on the worksheet and check answers or assign as homework and discuss the next day.

**Summary:** You have learned how the story behind the Seven Stars in the Crow culture and how those stars compare to the brightest stars as seen from Earth. You should now be able to use logarithmic formulas to calculate and simplify data and apply those formulas to real world situations.

**Materials/Resources Needed:**

- Worksheets for each student.
- Copy of the Seven Stars story, available in The Stars We Know by Timothy P. McCleary, pgs. 69-71, or in PDF form. (Available for purchase at <http://www.waveland.com/Titles/McCleary.htm>)
- Scientific calculators for students.
- For more information about astronomy in general and sky maps, go to <http://www.stargazing.net/david/index.html>.

**Extension:** This lesson could be extended by have students research the actual size of different stars and determine if the size of a celestial body affects magnitude and luminosity, and if so, with what type of relationship (linear, exponential, logarithmic, etc.).

# STUDENT WORKSHEET #1

Star Distance		in Light Years	Distance in Parsecs	Apparent Magnitude	Absolute Magnitude	Brightness
Big Dipper or Seven Stars	Alioth					
	Alkaid	100		1.85		
	Dubhe	120		1.87		
	Mizar	78		2.27		
	Alcor	82		3.99		
	Merak	79		2.35		
	Phecda	84		2.43		
	Megrez	81		3.3		
Polaris		430		2.01		
Sirius		8.6		-1.47		
Sun		.000016		-26.73		

## Formulas:

$$m = 6 - 2.5 \log \frac{L}{L_0}$$

$m$ —apparent magnitude  
 $L$ —luminosity of star  
 $L_0$ —luminosity of a sixth magnitude star (constant)

$$M = m + 5 - 5 \log d$$

$M$ —absolute magnitude

$$d = \frac{l}{3.262}$$

$d$ —distance from earth, in parsecs  
 $l$ —distance from earth, in light years

$$b = 2.512^{4.8-M}$$

$b$ —brightness of a star as compared to the sun

## Vocabulary:

*apparent magnitude:* The degree of brightness of a celestial body designated on a numerical scale, on which the brightest star (after the sun) has magnitude -1.4 and the faintest visible star has magnitude 6.

→ This rates how visible a celestial body is from Earth.

*absolute magnitude:* The magnitude of a star as it would appear to a hypothetical observer at a distance of 10 parsecs or 32.6 light-years.

→ This rates how visible celestial bodies are when they are all viewed from the same distance.

*luminosity:* The brightness of a star in comparison with that of the sun.

*parsec:* A unit of astronomical length based on the distance from Earth at which stellar parallax is 1 second of arc; equivalent to 3.262 light years.

*light year:* The distance that light travels in a vacuum in 1 year; 5.88 trillion miles or 9.46 trillion kilometers.

● Sirius/Bright Star/Morning Star

Big Dipper/Seven Stars

Dubhe ●

Merak ●

● Polaris/Star That  
Does Not Move

Phecda ●

Megrez ●

● Alioth

Alcor ● Mizar

● Alkaid

STUDENT WORKSHEET #2

1. The star Alioth is approximately 101 times brighter than the sun. What is its absolute magnitude?
2. If Alioth's apparent magnitude is 1.76, how far from Earth is it in terms of parsecs and light years?
3. Luminosity comparisons between other celestial bodies can be done by finding the difference in apparent magnitudes. Showing all steps, find how much brighter Polaris is than Sirius.
4. Using the formulas given, write one formula for finding apparent magnitude when given absolute magnitude and distance in light years.

TEACHER WORKSHEET #1

Star Distance		in Light Years	Distance in Parsecs	Apparent Magnitude	Absolute Magnitude	Brightness
Big Dipper or Seven Stars	Alioth	82 25		1.76	-.21	101
	Alkaid	100	31	1.85	-.58 142	
	Dubhe	120	37	1.87	-.96 201	
	Mizar	78	24	2.27	.38 59	
	Alcor	82	25	3.99	2 13	
	Merak	79	24	2.35	.43 56	
	Phecda	84	26	2.43	.38 59	
	Megrez	81	25	3.3	1.33 24	
Polaris		430	132	2.01	-3.59 2271	
Sirius		8.6	3	-1.47	1.4 23	
Sun		.000016	.00000494	-26.73	4.8 1	

**Formulas:**

$$m = 6 - 2.5 \log \frac{L}{L_0}$$

$m$ —apparent magnitude  
 $L$ —luminosity of star  
 $L_0$ —luminosity of a sixth magnitude star (constant)

$$M = m + 5 - 5 \log d$$

$M$ —absolute magnitude

$$d = \frac{l}{3.262}$$

$d$ —distance from earth, in parsecs  
 $l$ —distance from earth, in light years

$$b = 2.512^{4.8-M}$$

$b$ —brightness of a star as compared to the sun

**Vocabulary:**

*apparent magnitude:* The degree of brightness of a celestial body designated on a numerical scale, on which the brightest star (after the sun) has magnitude -1.4 and the faintest visible star has magnitude 6.  
 → This rates how visible a celestial body is from Earth.

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 → This rates how visible celestial bodies are when they are all viewed from the same distance.

*luminosity:* The brightness of a star in comparison with that of the sun.

*parsec:* A unit of astronomical length based on the distance from Earth at which stellar parallax is 1 second of arc; equivalent to 3.262 light years.

*light year:* The distance that light travels in a vacuum in 1 year; 5.88 trillion miles or 9.46 trillion kilometers.

TEACHER WORKSHEET #2

1. The star Alioth is approximately 101 times brighter than the sun. What is its absolute magnitude?

$$b = 2.512^{4.8-M}$$

$$101 = 2.512^{4.8-M}$$

$$\log(101) = \log(2.512^{4.8-M})$$

$$2 = (4.8 - M) * \log(2.512)$$

$$2 = (4.8 - M) * 0.4$$

$$5 = 4.8 - M$$

$$-0.2 = M$$

2. If Alioth's apparent magnitude is 1.76, how far from Earth is it in terms of parsecs and light years? Round final answers to the nearest whole number.

$$M = m + 5 - 5 \log d$$

$$-0.2 = 1.76 + 5 - 5 \log d$$

$$-6.96 = -5 \log d$$

$$1.392 = \log d$$

$$10^{1.392} = 10^{\log d}$$

$$25 = d$$

$$d = \frac{l}{3.262}$$

$$25 = \frac{l}{3.262}$$

$$82 = l$$

3. Luminosity comparisons between other celestial bodies can be done by finding the difference in apparent magnitudes. Showing all steps, find how much brighter Polaris is than Sirius. Round final answer to the nearest whole number.

METHOD 1

$$m_P - m_S = \left( 6 - 2.5 \log \frac{L_P}{L_0} \right) - \left( 6 - 2.5 \log \frac{L_S}{L_0} \right)$$

$$3.48 = -2.5 \log \frac{L_P}{L_S}$$

$$m_P - m_S = -2.5 \log \frac{L_P}{L_0} + 2.5 \log \frac{L_S}{L_0}$$

$$1.392 = \log \frac{L_P}{L_S} \text{ (drop the negative)}$$

$$m_P - m_S = -2.5 \left( \log \frac{L_P}{L_0} - \log \frac{L_S}{L_0} \right)$$

$$10^{1.392} = 10^{\log \frac{L_P}{L_S}}$$

$$m_P - m_S = -2.5 \left( \log \frac{L_P / L_0}{L_S / L_0} \right) = -2.5 \log \frac{L_P}{L_S}$$

$$25 = \frac{L_P}{L_S}$$

$$2.01 - (-1.47) = -2.5 \log \frac{L_P}{L_S}$$

## METHOD 2

$$m = 6 - 2.5 \log \frac{L}{L_0}$$

*POLARIS*

*SIRIUS*

$$2.01 = 6 - 2.5 \log \frac{L_P}{L_0}$$

$$-1.47 = 6 - 2.5 \log \frac{L_S}{L_0}$$

$$-3.99 = -2.5 \log \frac{L_P}{L_0}$$

$$-7.47 = -2.5 \log \frac{L_S}{L_0}$$

$$1.596 = \log \frac{L_P}{L_0}$$

$$2.988 = \log \frac{L_S}{L_0}$$

$$10^{1.596} = 10^{\log \frac{L_P}{L_0}}$$

$$10^{2.988} = 10^{\log \frac{L_S}{L_0}}$$

$$39 = \frac{L_P}{L_0}$$

$$973 = \frac{L_S}{L_0}$$

$$\text{ratio} = 973 \div 39 = 25$$



*Mathematics Grade 12 – Seven Stars (continued)*

4. Using the formulas given, write one formula for finding apparent magnitude when given absolute magnitude and distance in light years.

$$M = m + 5 - 5 \log d$$

$$-m = -M + 5 - 5 \log d$$

$$m = M - 5 + 5 \log d$$

$$m = M - 5 + 5 \log \frac{l}{3.262}$$

$$m = M - 5 + 5(\log l - \log 3.262)$$

$$m = M - 5 + 5 \log l - 5 \log 3.262$$

$$m = M - 7.567 + 5 \log l$$



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